Adaptive Modulation for MIMO Systems with Imperfect Channel Knowledge

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***Abstract*—Analytical studies on the potential capacity of MIMO systems inspire the development of new algorithms that decompose the MIMO channel into independent subchannels and adaptively control power and modulation level on these subchannels. Current research of adaptive modulation applying to MIMO systems assumes perfect channel knowledge at the receiver and ideal feedback from receiver to transmitter. In this paper, the impact of imperfect channel knowledge and feedback quantization error on adaptive modulation in MIMO systems are investigated. We evaluate the system BERs with channel estimates from training sequences. Rate-distortion theory is involved to assess the achievable performance of feedback quantization. Our simulation results show that, for a (4,4) MIMO system in a rich scattering environment and BER 10^{-3} level, compared to the case of perfectly known channel, 3 bits/vector symbol degradation in spectral efficiency occurs for a training length equal to four times the number of transmitter antennas. The results also show that it is sufficient to use 22 quantization bits per channel gain for feedback in a (4,4) system at BER 10^{-3}.**

I. Introduction

Analytical studies on multiple-input multiple-output (MIMO) Shannon capacity maximization for continuous input signals suggest that, if both the transmitter and receiver have perfect knowledge of the channel gain matrix and interference statistics, we can decouple the MIMO channel into independent subchannels and apply optimum water-filling on these subchannels [1][2], i.e., we allocate more power to subchannels with high channel gains. However, for finite alphabet input signals, we need to apply not only power allocation, but also other techniques, including varying the size of signal constellations. That is, for good subchannels, we allocate more power and use higher-level modulation. We refer joint adaptation of power and modulation level as adaptive modulation.

Adaptive modulation for MIMO systems maximizing information rate has been considered in [3]-[7] assuming perfect channel knowledge at both the transmitter and receiver. In practical systems, the channel transfer matrix and interference statistics are estimated by the receiver; hence perfect channel knowledge may not be obtainable. On the other hand, to enable adaptive modulation, the receiver has to feed channel information back to the transmitter in frequency division duplex (FDD) systems. In practical systems, due to limited bandwidth of the feedback channel, low-resolution quantized channel information is obtained at the transmitter [8]. In this paper, adaptive modulation with imperfect channel knowledge at transmitter and receiver is described. To separate the impact of imperfect channel estimates from that of imperfect feedback, in Section II, we focus on the performance of adaptive modulation using channel estimates from training sequences while assuming a perfect feedback path from the receiver to transmitter. In Section III, assuming perfectly known channel knowledge at the receiver, we investigate the effect of feedback quantization error where the achievable performance of quantization is assessed by rate-distortion theory [8]. In our work, we restrict the modulation schemes to BPSK, QPSK, 16-square QAM and 64-square QAM.

II. Effect of Imperfect Channel Estimates

Consider a MIMO link with N_t transmitting and N_r receiving antennas, and the received signal at time i is

$$\boldsymbol{y}_i = \boldsymbol{H}\boldsymbol{x}_i + \underbrace{\sqrt{\frac{P_I}{L}}\boldsymbol{H}_I\boldsymbol{b}_i + \boldsymbol{w}_i}_{\boldsymbol{n}_i}$$

where $\boldsymbol{x}_i$ is the transmitted vector symbol with power constraint P_s, $\boldsymbol{H}$ is the $N_r \times N_t$ quasi-static channel matrix of the desired user, and $\boldsymbol{n}_i$ is the interference plus noise vector. We assume that interference comes from L transmitters which could belong to one or more interferers. The total interference power is P_I, the $N_r \times L$ matrix $\boldsymbol{H}_I$ consists of quasi-static interfering users' channel gains, $\boldsymbol{b}_i$ consists of L independent zero-mean interfering signals each with unit variance, and $\boldsymbol{w}_i$ is the zero-mean additive white Gaussian noise vector with covariance matrix $\sigma^2\boldsymbol{I}$. Note that each interfering transmitter has the same transmitted power. It can be shown that the covariance matrix of interference plus noise is $\boldsymbol{R} = P_I/L\boldsymbol{H}_I\boldsymbol{H}_I^\dagger + \sigma^2\boldsymbol{I}$.

For a training length of N, denote the transmitted training vector symbols as $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N$ and the received vector signals as $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_N$. The maximum-likelihood (ML) estimates of the desired user's channel matrix and the spatial interference correlation matrix, $\hat{\boldsymbol{H}}$ and $\hat{\boldsymbol{R}}$, can be expressed as [9]

$$\hat{\boldsymbol{H}} = \boldsymbol{R}_{xy}^\dagger \boldsymbol{R}_{xx}^{-1}$$

and

$$\hat{\boldsymbol{R}} = \boldsymbol{R}_{yy} - \hat{\boldsymbol{H}}\boldsymbol{R}_{xy}$$

where the sample correlation matrices $\boldsymbol{R}_{yy} = \frac{1}{N}\sum_{i=0}^{N-1}\boldsymbol{y}_i\boldsymbol{y}_i^\dagger$, $\boldsymbol{R}_{xy} = \frac{1}{N}\sum_{i=0}^{N-1}\boldsymbol{x}_i\boldsymbol{y}_i^\dagger$, and $\boldsymbol{R}_{xx} = \frac{1}{N}\sum_{i=0}^{N-1}\boldsymbol{x}_i\boldsymbol{x}_i^\dagger$.

Assuming a perfect feedback path from the receiver to transmitter, adaptive modulation with channel estimates $\hat{\boldsymbol{H}}$ and $\hat{\boldsymbol{R}}$ is shown in Fig. 1. The pre- and post-processing matrices, obtained via singular value decomposition of $\hat{\boldsymbol{R}}^{-1/2}\hat{\boldsymbol{H}} = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{V}}^{\dagger}$, try to decompose the MIMO channel into subchannels with channel estimates. The vector signal to be conveyed, $\boldsymbol{s}_i$, has independent components. For a predefined target BER, the power and modulation level of each signal in $\boldsymbol{s}_i$ are determined from the diagonal matrix $\hat{\boldsymbol{\Lambda}}$ by an adaptive modulation algorithm used for OFDM systems [10]. To detect the transmitted signals, the receiver scales the appropriate constellations by the power coefficients from the adaptive modulation algorithm and the elements in $\hat{\boldsymbol{\Lambda}}$.

Monte Carlo simulation is used to evaluate the effect of imperfect channel estimates for different training lengths. We consider the desired user has a (4,4) antenna array and $L = 5$ interfering transmitters. Both the desired and interfering users are assumed to experience i.i.d. Rayleigh fading. We set the total interference power $P_I = 1$ and interference-to-noise power ratio INR=20dB. Due to the imperfect channel knowledge, the actual achieved BER is lower than the target BER. After experiments, it is found that to make the actual BER be 10^{-3} at SIR=15dB for training lengths $6N_t$ and $4N_t$, we have to set the target BERs to be 5×10^{-6} and 10^{-8}, respectively.

Fig. 2 shows the spectral efficiency for adaptive modulation with channel estimates where the actual BERs achieve 10^{-3}. As expected, the throughput improves as channel estimates become more accurate. Compared to the case of perfectly known channels, for a fixed SIR, about 3 bits/vector symbol degradation occurs with training length $4N_t$.

The spectral efficiency shown in Fig. 2 does not take the training overhead into account. Although a longer training length yields a higher throughput for data transmission period, it requires more overhead. To assess the overall spectral efficiency, we assume that the data is transmitted frame by frame, and that the channel is estimated at the beginning of each frame. For example, at SIR 15 dB, Fig. 2 shows that for training lengths $4N_t$ and $6N_t$, we can transmit 10 bits/vector symbol and 11 bits/vector symbol, respectively. With $N_t = 4$ transmitting antennas and a frame length M, the amount of information transmitted in one frame can be calculated as $(M - 4 \times N_t) \times 10$ bits and $(M - 6 \times N_t) \times 11$ bits for training lengths $4N_t$ and $6N_t$, respectively. It can be shown that if the frame length is more than 104 vector symbols, it is worth using training length $6N_t$.

III. Effect of Feedback Quantization

In practice, transmitter and receiver may not have the same channel information due to feedback quantization error. To enable the analysis of quantization error, we assume perfect channel knowledge at the receiver. A noise-limited environment with negligible interference and spatially white Gaussian noise is considered. Independent Rayleigh fading is assumed for the desired user where channel matrix $\boldsymbol{H}$ has $N_r N_t$ i.i.d. zero-mean circularly symmetric complex Gaussian components with unit variance. Equivalently, with real and imaginary parts, $\boldsymbol{H}$ has $2N_r N_t$ i.i.d. zero-mean real Gaussian random variables each with variance $\sigma_H^2 = 1/2$. Assume that N_q bits are used to vector-quantize the $2N_r N_t$ elements of $\boldsymbol{H}$. Letting $\tilde{\boldsymbol{H}}$ be the quantized $\boldsymbol{H}$, we consider vector quantization that minimizes mean squared error [8]

$$\sigma_\epsilon^2 = \frac{1}{2N_r N_t} \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} \sum_{k \in \{\text{real,imag}\}} E\left[(H_{ijk} - \tilde{H}_{ijk})^2 \right]$$

where H_{ijk} and $\tilde{H}_{ijk}$ denote the real-valued elements of matrix $\boldsymbol{H}$ and $\tilde{\boldsymbol{H}}$, respectively. Since the real and imaginary components in $\boldsymbol{H}$ are i.i.d. Gaussian, the mean squared error can be lower-bounded by the corresponding distortion-rate function [11], i.e.,

$$\sigma_\epsilon^2 \geq D(R_q) = \sigma_H^2 2^{-2R_q} = \frac{1}{2} 2^{-2R_q} \tag{1}$$

where $R_q = N_q/(2N_r N_t)$ is the number of descriptive bits per real component of $\boldsymbol{H}$. For a fixed R_q, this lower bound can be approached arbitrarily closely as $2N_r N_t$ goes to infinity. For a very large $2N_r N_t$, this lower bound is achieved when the quantization error $\boldsymbol{\epsilon} = \boldsymbol{H} - \tilde{\boldsymbol{H}}$ is a zero-mean complex Gaussian random matrix, independent of $\tilde{\boldsymbol{H}}$, with i.i.d. real and imaginary parts each having variance $D(R_q)$. More specifically, for $\epsilon_{ijk} = H_{ijk} - \tilde{H}_{ijk}$, the lower bound of quantization error is achieved when $\epsilon_{ij} \sim \mathcal{N}(0, D(R_q))$ and $\tilde{H}_{ijk}$ is independent of ϵ_{ijk}. It is obvious that $\tilde{H}_{ijk} \sim \mathcal{N}(0, \sigma_H^2 - D(R_q))$. It can also be shown that H_{ijk} and $\tilde{H}_{ijk}$ are jointly Gaussian random variables with covariance $\text{cov}(H_{ijk}, \tilde{H}_{ijk}) = \sigma_H^2 - D(R_q)$.

For the adaptive modulation with quantized feedback, the transmitter determines the the pre-processing matrix and adaptive modulation according to the quantized channel matrix; while the receiver uses the perfect known channel matrix to determine the post-processing matrix and demodulation the signal.

Monte Carlo simulation is used to evaluate the effect of feedback quantization on the performance of adaptive modulation. To simulate matrices $\boldsymbol{H}$ and $\tilde{\boldsymbol{H}}$, i.i.d. pairs of $(H_{ijk}, \tilde{H}_{ijk}), i = 1, \ldots, N_r, j = 1, \ldots, N_t, k \in$ {real,imag} are generated. For each pair, H_{ijk} and $\tilde{H}_{ijk}$ are jointly real Gaussian with

$$\begin{pmatrix} H_{ijk} \\ \tilde{H}_{ijk} \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} 0.5 & 0.5 - D(R_q) \\ 0.5 - D(R_q) & 0.5 - D(R_q) \end{bmatrix} \right)$$

where $D(R_q)$ is obtained from the analysis leading to (1).

We consider a $(4,4)$ MIMO system with SNR 15 dB. Fig. 3 shows the upper bound of BER performance (best possible BER) versus the number of quantization bits with target BER 10^{-3}. In Fig. 3, we observe that the BER improves significantly as the number of quantization bits increases, and eventually reaches the target BER. We observe that 22 bits/complex channel gain should be used for the feedback quantization. Hence, for a $(4,4)$ MIMO link, with target BER=10^{-3} and SNR=15dB, we need $22 \times 16 = 352$ bits to quantize the channel matrix. Since the receiver has four antennas, ignoring channel coding and assuming that each antenna uses QPSK, we need $352/(4 \times 2) = 44$ vector symbols for the feedback of channel matrix.

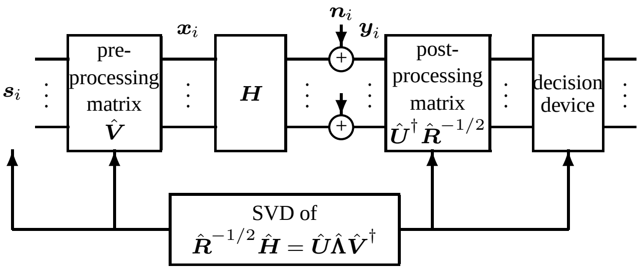

Fig. 1. Adaptive modulation with imperfect channel estimates.

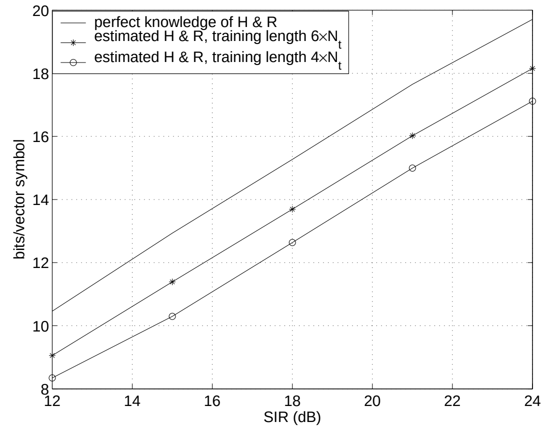

Fig. 2. Number of bits transmitted per vector symbol versus SIR for imperfect channel estimates with $N_t = N_r = 4$ and $L = 5$. The actual achieved BER is 10^{-3}.

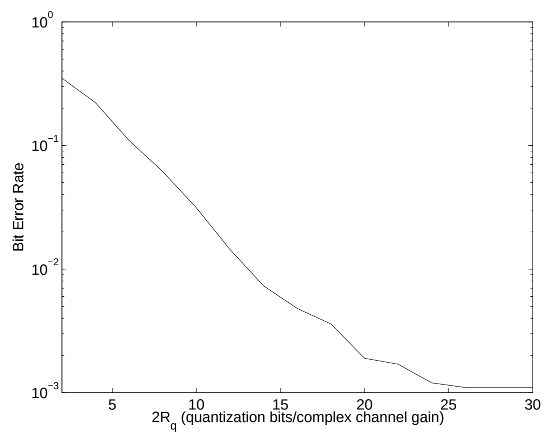

Fig. 3. Average BER versus the number of quantization bits of feedback with $N_t = N_r = 4$, SNR=15dB and target BER 10^{-3}.

To see how much overhead is required due to the feedback of channel matrix, we consider a system operating at a carrier frequency of 1.9 GHz and symbol rate 24.3 ksymbol/sec in a bandwidth of 30 kHz [12]. If the mobile is moving at $v = 5$ m/sec (18 km/hour), the maximum Doppler shift $f_m = v/\lambda = 31.7$ Hz where λ is the wavelength at the carrier frequency. The normalized Doppler spread $f_m/$symbol rate $= 0.13\%$, which can also be interpreted as the ratio of symbol duration to channel coherence time. Assuming that the frame length is 200 vector symbols (the assumption of quasi-static channel holds for normalized Doppler spread 0.13%), for the $(4,4)$ MIMO link and target BER 10^{-3} discussed above, the overhead for feedback is $44/200 = 22\%$.

IV. Conclusions

In this paper, we have investigated the effects of imperfect channel estimates and feedback quantization error on adaptive modulation for MIMO systems. In studying the effect of imperfect channel estimates, we assumed a perfect feedback path from receiver to transmitter. Compared to the case of perfectly known channels, for a $(4,4)$ MIMO system and a rich scattering environment, 3 bits/vector symbol degradation in spectral efficiency occurs for a training length equal to four times the number of transmitting antennas and the actual achieved BER 10^{-3}. In analyzing the effect of feedback quantization error, we assume perfect channel knowledge at the receiver and a noise-limited environment. It is shown that for a $(4,4)$ MIMO link with independent Rayleigh fading, to achieve target BER 10^{-3}, 22 bits/complex channel gain should be used for the feedback quantization, which may be translated into the required overhead for feedback. The combined effect of imperfect channel estimates and imperfect feedback is left for future work.

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