

# Cross-layer optimization of rateless coding over wireless fading channels

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**Abstract**—Rateless codes are recently-proposed erasure correction codes. To apply rateless codes over wireless communication channels, a physical-layer forward error correction (FEC) code, such as a convolutional code, is usually used to correct errors within each packet while Raptor codes are used in the application layer to correct erased packets. Traditionally, the physical-layer modulation and coding rate are chosen to guarantee an overall packet error rate to be below a certain level. However, such a choice does not always provide the best overall system performance. This paper proposes a cross-layer scheme to optimize physical layer modulation and coding rate to maximize system throughput. Both slow and fast fading channels are considered. For slow fading channels, cross-layer adaptive modulation and coding schemes are also proposed. Numerical results show that the proposed cross-layer schemes outperform traditional schemes significantly in terms of system throughput. The results also indicate that in many situations, allowing for more packet error correction in the application-layer through erasure codes can be more efficient than ensuring a low packet error rate using a low-rate physical-layer code.

**Index Terms**—Rateless codes, adaptive modulation and coding, cross-layer design.

## I. INTRODUCTION

Rateless codes, also known as fountain codes, have been recently proposed for powerful erasure forward error correction (FEC). Rateless codes can continuously generate potentially unlimited numbers of data streams until an acknowledgement from the receiver is received indicating successful decoding. Raptor codes [1], as a successful implementation of rateless codes, have been standardized as application layer FECs for file downloading services in the third generation partnership program (3GPP) Multimedia Broadcast/Multicast Services (MBMS) [2].

To date, most studies of Raptor codes focus on memoryless erasure channels where the erasure rate is fixed and known. However, due to the time varying nature of wireless channels, a physical-layer code is required to provide protection against fading and noise. In most communication systems, the physical-layer code and application-layer erasure code are studied separately. The quality of service (QoS) is usually guaranteed by the physical layer code to ensure a packet error rate (PER) below a certain level (1 percent for example). The packets that fail to be decoded by the physical-layer code are then corrected at the packet level by an erasure FEC or retransmitted using an Automatic Retransmission reQuest (ARQ) protocol. This scheme usually results in very little protection at the packet level. From an information theory

perspective, since one can always use an ideal physical layer code to drive the packet error rate to zero, erasure protection at the packet-level is not needed. If such an ideal physical layer code could be designed at a rate nearly equal to the information capacity that the channel supports, then a scheme without packet level erasure protection would be indeed “optimum”. However, in practice, such an ideal physical-layer code does not exist, especially for time varying fading channels.

As the system simulation results in [2] suggest, in a Raptor coded broadcast system, a tradeoff exists between physical and application-layer code rates. In many cases, a higher PER that is corrected by the application-layer Raptor code can be more efficient than traditional designs. However, the results provided in [2] are from system simulations, and do not provide sufficient insight and quantification of the optimal rate tradeoffs. In [4], we also investigate this tradeoff by evaluating the capacity of Raptor codes in hybrid error-erasure channels when different physical layer code rates are used (see Fig. 4 of [4]), but do not attempt to find the optimal rate tradeoff. Recently, [3] discusses certain aspects of the tradeoff between physical-layer code rate and packet-level erasure code rate in Rayleigh block fading channels. However, the formulation in [3] considers transmitting a prescribed number of information bits within a given transmission time. The rate of the packet-level rateless code is fixed during the whole transmission period. The objective of the optimization is to minimize the operational channel average SNR under certain outage constraints. The analysis in [3] provides great theoretical insights of the physical layer and application layer rate tradeoff. However, in systems where the objective is to maximize the throughput, the optimal solution in [3] can be very inefficient in practice if the actual channel average SNR is known and much higher than the minimum operational SNR. In addition, [3] considers an ideal physical-layer code which gives a zero packet error rate as long as the code rate is below the information capacity that the channel supports. Finally, [3] also does not consider rate adaption in the physical-layer when the channel fading is slow.

In this paper, we investigate optimization of the QoS tradeoff between the physical-layer and the application-layer for rateless coded systems in fading channels. The application-layer rateless code is assumed to keep generating coded packets on the fly until all erroneous packets are corrected. Therefore, the “rateless” property is exploited in our scheme to drive outage to nearly zero; while in [3], the erasure code rate is fixed during transmission and there is a certain probability of outage. In addition, compared to [3], more practical error

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curves for physical layer codes are used. We consider Rayleigh fading channels with both slow and fast fading. For fast fading channels, we find the optimal physical-layer modulation and code rate that maximizes the overall system throughput. We compare the proposed choice of the physical-layer code rate to the traditional choice of the physical-layer code rate to demonstrate the advantage of the proposed optimization method.

For slow fading channels where physical-layer rate adaptation is feasible, we propose a cross-layer adaptive modulation and coding scheme which maximizes overall system throughput. Adaptive modulation and coding (AMC) (e.g [5]) have been proposed for the physical layer in the literature for many different communication scenarios. In [6], AMC have been used in combination with truncated ARQ. However, all existing adaptive schemes essentially choose the modulation and coding mode to guarantee that the PER does not exceed a certain level rather than provide the best overall system performance. We derive the overall system throughput for different scenarios and compare the proposed cross-layer AMC design to traditional non cross-layer AMC designs.

The rest of the paper is organized as follows: Section II describes the system setup, channel models, as well as introduces the performance measures in application-layer Raptor coding and physical-layer modulation and coding. Section III discusses and derives the throughput performance of the proposed cross-layer transmission in fast fading channels as well as the cross-layer AMC design in slow fading channels. Section IV presents the numerical results, which includes a comparison between the proposed cross-layer design and traditional non cross-layer design.

## II. SYSTEM SETUP AND CHANNEL MODELS

### A. System model

The paper considers a two-layer model where the transmitter attempts to deliver messages to single or multiple users. The information bits are divided into data frames. Each data frame contains a total of  $K$  source symbols which are encoded by a Raptor code to generate a potentially infinite number of Raptor encoded symbols. Each source symbol and a Raptor-encoded symbol contains  $S_R$  bits.  $L_R$  Raptor encoded symbols form a packet together with packet header information ( $P_h$  bits) and Cyclic Redundancy Checks (CRC) ( $P_{CRC}$  bits). Each packet is further protected by a physical-layer code with a code rate  $R_c$  and modulated using  $M$ -QAM and transmitted to the wireless fading channel. Finally, pilot symbols are added to every one or more code frames depending on how fast the channel fades. The structure of each packet is shown in Fig. 1. For each data frame, the transmitter keeps sending packets until either it receives acknowledgements from all the intended receivers or until the maximum number of packets allowed has been reached. After that, the transmitter starts to transmit the next data frame. On the receiver side, each user first demodulates and decodes the physical-layer code and checks its correctness with a CRC. If there are uncorrected errors, the entire packet is dropped. Otherwise, the correctly decoded packets are further used to decode the original information data using a Raptor decoder. Once the receiver successfully decodes the data frame, an acknowledgement is sent to the transmitter. Let  $N_p$  be the actual number of packets transmitted for each

data frame. Then the number of Raptor encoded symbols sent is  $K_t = N_p L_R$ . For analytical simplicity, we assume  $L_R = 1$ , i.e., each Raptor coded symbol is a physical-layer packet.

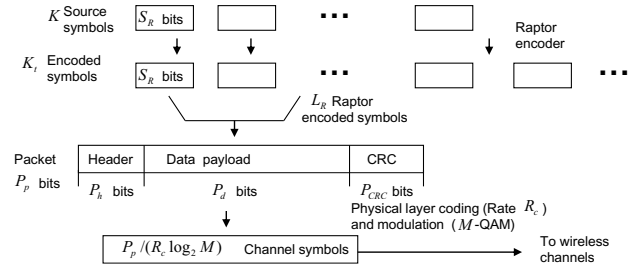


Fig. 1. System setup and packet structure.

To quantify the overall system performance, we calculate the overall system throughput in terms of the average number of information bits transmitted per channel symbol. To simplify the notation, define  $R_R = K/K_t$  as the realized Raptor code rate,  $\varepsilon_p = (P_{CRC} + P_h)/P_d$  as the packet overhead where  $P_h, P_{CRC}$  and  $P_d = L_R S_R$  are the numbers of bits representing packet header information, CRCs, and data payload in each physical-layer packet, respectively.  $m = \log_2 M$  is the modulation rate. Then the system throughput can be calculated as

$$TP = \frac{K S_R}{((P_h + P_{CRC} + P_d)N / (m R_c))} = \frac{1}{1 + \varepsilon_p} m R_c R_R. \quad (1)$$

### B. Physical-layer channel model

We assume the channel fading between the server and all the users to be independently and identically distributed (i.i.d) Rayleigh block fading channels. The channel quality of each user is characterized by the instantaneous signal-to-noise-ratio (SNR)  $\gamma$ . In the Rayleigh fading model, the probability density function (pdf) of the instantaneous channel SNR  $\gamma$  can be characterized as:

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \quad (2)$$

where  $\bar{\gamma}$  is the average channel SNR.

We also categorize the channel fading into two different scenarios depending on the coherence time of channel fades. In the fast fading scenario, we assume that the coherence time of the channel fades are of a similar length to the packet transmission time. Therefore, the instantaneous channel SNR remains the same for each packet, but varies from packet to packet. In the slow fading scenario, we assume the coherence time to be much longer than the transmission time of a Raptor coded data frame. Therefore, for slow fading, the instantaneous SNR is assumed to remain the same for the whole Raptor coded data frame but varying from frame to frame. It is also assumed that the instantaneous channel SNR is available to the transmitter in the slow fading scenario.

### C. Packet error rate and physical layer design

At the physical layer, we consider possible choices of multiple transmission modes with different combinations of

modulation and convolutional coding pairs as in [6], which are borrowed from HIPERLAN/2, IEEE 802.11a and 3GPP standards. For analytical simplicity, we use the PER expressions developed in [6], where for a given modulation and code pair in mode  $n$ , the packet error rate is approximated by:

$$PER_n(\gamma) = \begin{cases} 1 & \text{if } \gamma < \gamma_{pn} \\ a_n \exp(-g_n \gamma) & \text{if } \gamma \geq \gamma_{pn} \end{cases} \quad (3)$$

where  $\gamma$  is the instantaneous channel SNR, which is assumed to remain unchanged for the whole packet. In [6],  $a_n, g_n$  and  $\gamma_{pn}$  are parameters that are mode-dependent, the values of which for different uncoded M-QAM (rectangular) modulations (TM1) and convolutionally coded modulations (TM2) are listed in Table 1 and Table 2 of [6], respectively.

#### D. Raptor codes in application layer

Raptor codes see a virtual erasure channel in the application layer. The erasure probability  $P_e$  is equal to the packet error rate. The inefficiency of the Raptor code used in the 3GPP standard can be well modeled by the following equation [2],

$$P_{fail}(K_s, K) = \begin{cases} 1 & \text{if } K_s \leq K \\ a \times b^{K_s - K} & \text{if } K_s > K \end{cases} \quad (4)$$

where  $P_{fail}(K_s, K)$  denotes the probability that the receiver fails to decode  $K$  source packets after  $K_s$  packets are successfully received,  $a = 0.85$  and  $b = 0.567$  are constants. Denote  $K_s$  and  $K_t$  as random variables representing the number of successfully received packets, and the total number of transmitted packets during the transmission of one Raptor code frame, respectively. Consider a continuous transmission of  $q$  frames. The effective Raptor code rate  $R_R = \frac{qK}{\sum_{i=1}^q K_t(i)}$ , where  $K_t(i)$  is the number of packets transmitted during  $i$ -th frame. As  $q$  gets large over a long time, according to the Weak Law of Large Numbers,  $\frac{1}{q} \sum_{i=1}^q K_t(i) \rightarrow E(K_t)$ . Therefore, the average long-term effective Raptor code rate can be evaluated as  $R_R = K/E(K_t)$ .

To evaluate  $E(K_t)$ , since the packets transmitted through the erasure channel represent a Bernoulli process with failure probability equal to the erasure probability  $P_e$ , it can be shown (proof-omitted) that the random variable  $K_t$ , conditioned on  $K_s$  follows a negative binomial distribution, with average value equal to  $E(K_t|K_s) = \frac{K_s}{1-P_e}$ . Hence  $E(K_t) = E(E(K_t|K_s)) = E(\frac{K_s}{1-P_e}) = \frac{E(K_s)}{1-P_e}$ . To calculate  $E(K_s)$  using Eq. (4), we have, for a positive integer  $x$ ,

$$\begin{aligned} P(K_s = K + x) &= Prob(\text{decoding failure when } K_s < K + x) \\ &\quad \times Prob(\text{decoding success when } K_s = K + x) \\ &= \prod_{i=0}^{x-1} (a \times b^i) \times (1 - a \times b^x); \end{aligned} \quad (5)$$

Therefore,

$$\begin{aligned} E(K_s) &= K + \sum_{x=0}^{\infty} x \times P(K_s = K + x) \\ &= K + \sum_{x=0}^{\infty} x \left[ \prod_{i=0}^{x-1} (a \times b^i) \times (1 - a \times b^x) \right] \\ &= K + \sum_{x=0}^{\infty} (x a^x \times b^{x(x-1)/2} - x a^x \times b^{x(x+1)/2}) \\ &= K + C, \end{aligned} \quad (6)$$

where  $C$  is a constant that can be easily evaluated numerically (here  $C \simeq 1.39$ ). Therefore, the realized Raptor code rate<sup>1</sup> can be evaluated as

$$R_R = \frac{K}{E(K_t)} = \frac{K(1 - PER)}{K + C} = \frac{(1 - PER)}{1 + \varepsilon_r}, \quad (7)$$

where  $\varepsilon_r = C/K$  is defined as the Raptor code overhead, which decreases as the Raptor code dimension  $K$  increases.

### III. PERFORMANCE ANALYSIS

#### A. Fast fading

In the fast fading scenario, each Raptor encoded packet experiences a different instantaneous SNR. Therefore the erasure rate for application layer Raptor codes is equal to the PER averaged over different channel SNRs. Therefore, for a given transmission mode  $n$  (with modulation rate  $m_n = \log_2 M$  and code rate  $R_{cn}$ ), using Eqs. (3) and (2), the erasure rate for the Raptor code is:

$$\begin{aligned} P_e(n) &= \int_0^{\infty} PER_n(\gamma) f_\gamma(\gamma) d\gamma \\ &= \int_0^{\gamma_{pn}} f_\gamma(\gamma) d\gamma + \int_{\gamma_{pn}}^{\infty} a_n \exp(-g_n \gamma) f_\gamma(\gamma) d\gamma \\ &= 1 - \exp\left(-\frac{\gamma_{pn}}{\bar{\gamma}}\right) + \frac{a_n}{\bar{\gamma} g_n + 1} \exp\left(-\left(g_n + \frac{1}{\bar{\gamma}}\right) \gamma_{pn}\right). \end{aligned} \quad (8)$$

By substituting Eqs. (7) and (8) into Eq. (1), the system throughput in fast fading for given transmission mode  $n$  is given by

$$\begin{aligned} TP_{fast}(n) &= \frac{1}{(1 + \varepsilon_p)(1 + \varepsilon_r)} m_n R_{cn} (1 - P_e(n)) \\ &= \frac{1}{(1 + \varepsilon_p)(1 + \varepsilon_r)} m_n R_{cn} \left( \exp\left(-\frac{\gamma_{pn}}{\bar{\gamma}}\right) \right. \\ &\quad \left. - \frac{a_n}{\bar{\gamma} g_n + 1} \exp\left(-\left(g_n + \frac{1}{\bar{\gamma}}\right) \gamma_{pn}\right) \right). \end{aligned} \quad (9)$$

**1) Traditional non cross-layer scheme:** Commonly, the physical layer modulation and coding design is independent of the upper layer design. The QoS is guaranteed entirely by the physical layer which ensures that the packet dropping rate is below a certain level. Therefore, when a traditional non cross-layer scheme is used, the transmitter chooses the transmission mode with the highest modulation and coding rate which guarantees that the average PER (or erasure rate) is below a

<sup>1</sup>We have assumed that the maximum number of transmitted symbols for each code frame is large enough except when  $PER = 1$ , in which case an outage event occurs.

certain value  $P_{loss}$ . Therefore, for a known average SNR, transmission mode  $n$  is chosen according to the following criterion:

$$n = \arg \max_n (m_n R_{cn}) \text{ s.t. } P_e(n) \leq P_{loss}. \quad (10)$$

We remark that  $P_{loss} = 0.01$  is chosen for all performance comparisons in this paper.

2) **Proposed cross-layer scheme:** In the proposed cross-layer scheme, the physical layer design is aware of the application layer FEC. Therefore, the QoS is guaranteed by both the physical layer and the application layer FECs. The application layer rateless code is able to correct dropped packets by generating sufficient numbers of Raptor encoded packets to drive the outage probability to nearly zero. Therefore, the objective of the proposed cross-layer design is to maximize the overall system throughput. Since the overhead parameters  $\varepsilon_p$  and  $\varepsilon_r$  are independent of mode  $n$ , the criteria for choosing the transmission mode  $n$  based on our proposed cross-layer scheme can be found by using Eq. (9):

$$\begin{aligned} n &= \arg \max_n \{TP_{fast}(n)\} \\ &= \arg \max_n \{m_n R_{cn} (\exp(-\frac{\gamma_{pn}}{\bar{\gamma}}) \\ &\quad - \frac{a_n}{\bar{\gamma}g_n + 1} \exp(-(g_n + \frac{1}{\bar{\gamma}})\gamma_{pn}))\} \end{aligned} \quad (11)$$

### B. Slow fading

When the fading is slow, the instantaneous SNR remains the same for each Raptor coded data frame. Therefore each coded frame essentially experiences an AWGN channel. By substituting Eq. (7) to Eq. (1), we obtain that the system throughput performance in an AWGN channel with SNR  $\gamma$  when mode  $n$  is chosen is given by:

$$TP_{AWGN}(n, \gamma) = \frac{m_n R_{cn} (1 - PER_n(\gamma))}{(1 + \varepsilon_p)(1 + \varepsilon_r)}, \quad (12)$$

where  $PER_n(\gamma)$  is given by Eq. (3).

1) **Non adaptive scheme:** When a non-adaptive scheme is used, the transmission mode  $n$  is fixed during the whole transmission period. The long term average throughput for a non-adaptive scheme using mode  $n$  is then given by:

$$TP_{slow}(n) = \int_0^\infty TP_{AWGN}(n, \gamma) f_\gamma(\gamma) d\gamma. \quad (13)$$

By substituting Eqs. (12) and (3) into Eq. (13), we obtain,

$$\begin{aligned} TP_{slow}(n) &= \frac{1}{(1 + \varepsilon_p)(1 + \varepsilon_r)} m_n R_{cn} (\exp(-\frac{\gamma_{pn}}{\bar{\gamma}}) \\ &\quad - \frac{a_n}{\bar{\gamma}g_n + 1} \exp(-(g_n + \frac{1}{\bar{\gamma}})\gamma_{pn})). \end{aligned} \quad (14)$$

2) **Proposed cross-layer AMC scheme:** For the slow fading scenario, we propose a cross-layer AMC scheme where the transmission mode is chosen according to the instantaneous SNR to maximize the system throughput. Therefore, for a given instantaneous channel SNR  $\gamma$ , the modulation and coding mode is chosen according to

$$n = \arg \max_n \{m_n R_{cn} (1 - PER_n(\gamma))\}. \quad (15)$$

The design criteria can be further simplified to choose mode  $n$  when the SNR  $\gamma$  lies between  $\gamma_n$  and  $\gamma_{n+1}$ , where  $\gamma_1 = \gamma_{p1}$ ,  $\gamma_{N+1} = \infty$  and  $\gamma_n (n = 1, 2, \dots, N)$  is the solution of the following equation,

$$\begin{aligned} (1 - a_{n-1} \exp(-g_{n-1}\gamma_n)) \times m_{n-1} R_{c(n-1)} \\ = (1 - a_n \exp(-g_n\gamma_n)) \times m_n R_{cn} \end{aligned} \quad (16)$$

for  $n = 1, 2, \dots, N$ , where we have assumed that the transmission mode is ordered such that  $m_n R_{cn}$  is monotonically increasing with  $n$ , and  $N$  is the number of available transmission modes. When the channel SNR is below  $\gamma_{p1}$ , the PER approaches 1, where an outage event occurs. The outage probability can be calculated as

$$P_{out} = \int_0^{\gamma_{p1}} f_\gamma(\gamma) d\gamma = 1 - \exp(-\frac{\gamma_{p1}}{\bar{\gamma}}). \quad (17)$$

The probability that mode  $n$  is chosen is given by:

$$P_n = \int_{\gamma_n}^{\gamma_{n+1}} f_\gamma(\gamma) d\gamma = \exp(-\frac{\gamma_n}{\bar{\gamma}}) - \exp(-\frac{\gamma_{n+1}}{\bar{\gamma}}). \quad (18)$$

Therefore, the long-term average system throughput using the proposed cross-layer AMC scheme can be evaluated as:

$$\begin{aligned} TP_{AMC} &= \sum_{n=1}^N P_n E(TP_{AMC} | \gamma_n < \gamma \leq \gamma_{n+1}) \\ &= \sum_{n=1}^N P_n \int_{\gamma_n}^{\gamma_{n+1}} TP_{AWGN}(n, \gamma) \frac{f_\gamma(\gamma)}{P_n} d\gamma \\ &= \sum_{n=1}^N \int_{\gamma_n}^{\gamma_{n+1}} TP_{AWGN}(n, \gamma) f_\gamma(\gamma) d\gamma. \end{aligned} \quad (19)$$

Applying Eqs. (2) and (12) to Eq. (19), we obtain the average throughput

$$\begin{aligned} TP_{AMC} &= \frac{1}{(1 + \varepsilon_h)(1 + \varepsilon_r)} \left\{ \sum_{n=1}^N m_n R_{cn} \times [\exp(-\frac{\gamma_n}{\bar{\gamma}}) \right. \\ &\quad - \exp(-\frac{\gamma_{n+1}}{\bar{\gamma}}) + \frac{a_n}{b_n \bar{\gamma}} \exp(-b_n \gamma_{n+1}) \\ &\quad \left. - \frac{a_n}{b_n \bar{\gamma}} \exp(-b_n \gamma_n) \right\}, \end{aligned} \quad (20)$$

where  $b_n = g_n + 1/\bar{\gamma}$ .

3) **Non cross-layer AMC scheme:** Traditionally, AMC design in physical layer is independent of data link layer and application layer designs. If AMC is used in a traditional non cross-layer design, the transmitter chooses the transmission mode with the highest modulation and coding rate while still ensuring the packet error rate given the current channel instantaneous SNR is below threshold  $P_{loss}$ . Therefore, the choice of the modulation and coding pair is according to the following criterion:

$$n = \arg \max_n (m_n R_{cn}) \text{ s.t. } PER_n(\gamma) \leq P_{loss}, \quad (21)$$

where  $PER_n(\gamma)$  is given by Eq. (3). The outage probability and overall throughput using a traditional non cross-layer AMC scheme can be obtained similarly except the threshold  $\gamma_n$  is

$$\gamma_n = \frac{1}{g_n} \ln\left(\frac{a_n}{P_{loss}}\right). \quad (22)$$

Note that the choice of the transmission mode of the non cross-layer AMC scheme described above is the same as that of the AMC design described in [6]. The only difference is that the threshold  $P_{target}$  is used in [6] instead of  $P_{loss}$  used in this paper, where  $P_{target}$  is a constant determined by the maximum number of re-transmission attempts allowed in ARQ protocol as well as by the maximum packet loss rate after the re-transmission attempts.

#### IV. NUMERICAL RESULTS

With a packet length  $P_p = 1080$  bits, the parameters  $a_n, g_n, \gamma_{pn}$  for calculating packet error rates under different modulation and coding modes have been listed in Table I and Table II of [6]. The throughput performance of the combinations of different modulation and coding schemes (uncoded and convolutionally coded) with erasure Raptor codes over AWGN channels can be calculated by Eqs. (12) and (3). The throughput performances of uncoded M-QAM modulations and convolutionally coded M-QAM modulations in AWGN channels are shown in Figs. 2 and 3, respectively. Although the focus of the paper is on fading channels, the results of AWGN channels are also presented to illustrate how different modulation and coding modes are chosen at different SNRs in adaptive schemes. From Figs. 2 and 3, it can be seen

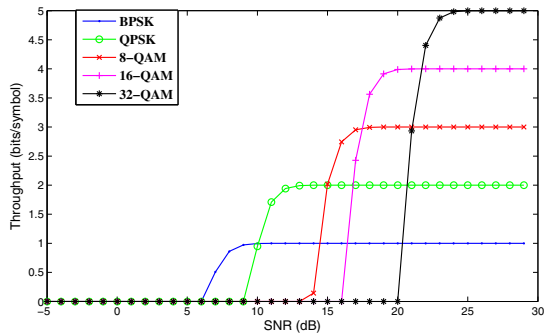


Fig. 2. Throughput performance of uncoded packets in AWGN channels.

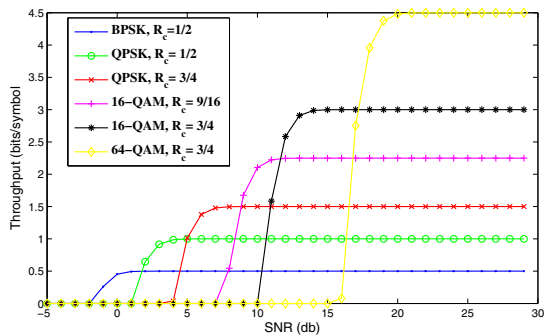


Fig. 3. Throughput performance of convolutionally coded packets in AWGN channels.

that the best modulation and coding mode which maximizes the throughput differs for different SNRs. In very low SNR,

coded BPSK offers the best throughput performance; while at high SNR, uncoded high level modulation is best. This is to be expected, as in low SNR, low-rate codes and modulation schemes should be chosen to achieve an acceptable packet error rate; On the other hand, in high SNR, a small packet error rate can be easily corrected by the Raptor codes, and a high-rate modulation and coding scheme can achieve a higher rate. Therefore, it is beneficial to use AMC to select the best modulation and coding mode according to the SNR when the channel varies over time. Note that the uncoded scheme using BPSK, QPSK, 8-QAM and 16-QAM does not offer maximum throughput at any SNR. Therefore, in the proposed AMC scheme for slow fading channels, only the coded modulation mode and uncoded 32-QAM, 64-QAM, 128-QAM transmission modes are candidates.

We next compare the performance of different choices of modulation and coding schemes in fast fading channels. Fig. 4 shows the throughput performance of the proposed cross-layer optimized transmission mode for Rayleigh fading channels for different average SNRs. For comparison, throughput performance of three specific transmission modes as well as the transmission mode chosen by the non cross-layer scheme are also shown in Fig. 4. For all the curves in Fig. 4, the throughput is calculated using Eq. (9), with the transmission mode  $n$  being selected according to the corresponding criteria described as follows: for the “proposed cross-layer selection scheme”, the transmission mode  $n$  is selected according to the criteria described by Eq. (11); for the traditional “non cross-layer selection scheme”, the transmission mode  $n$  is chosen according to the criteria described by Eq. (10); for the curves representing specific transmission mode  $n$ , the modulation and coding pair used for the transmission is pre-designed and fixed for all the average SNR values. It can be seen that the proposed cross-layer transmission scheme offers significantly better overall system performance over most of the SNR range compared to traditional non cross-layer transmission scheme.

Fig. 5 shows the resulting average packet error rate  $P_e(n)$  for the proposed cross-layer scheme and the traditional non cross-layer scheme in fast fading channels. The average PER is calculated using Eq. (8) with the transmission mode  $n$  selected according to the corresponding criteria. It can be observed that while traditional non cross-layer schemes use strong and conservative physical layer coding to guarantee a low PER, the optimal choice of transmission mode from a throughput maximization perspective involves transmission with a high PER to be corrected by the application-layer Raptor code.

In Fig. 6, for the slow fading scenario, the throughput performance of the proposed cross-layer AMC scheme is shown in comparison with the best performing non-adaptive scheme, as well as AMC using traditional non cross-layer design criteria. When the proposed cross-layer AMC scheme is used, the transmission mode  $n$  for any given instantaneous SNR  $\gamma$  is chosen according to the criteria described in Eq. (15). The throughput performance of the cross-layer AMC scheme is given by Eq. (20), where the threshold parameters  $\gamma_n$  is given by the solution of Eq. (16). When the non cross-layer AMC scheme is used, the transmission mode  $n$  for any given instantaneous SNR  $\gamma$  is selected according to criteria described by Eq. (21) and the throughput of non cross-layer AMC scheme is calculated using Eq (20) with parameters

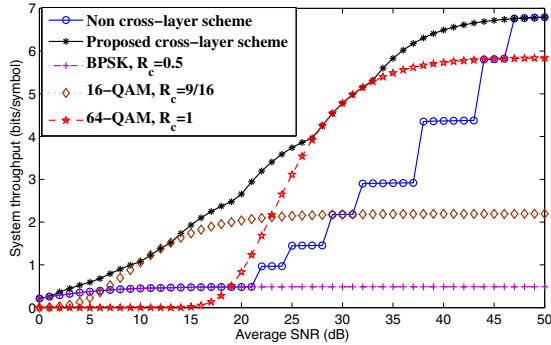


Fig. 4. Throughput performance comparison for fast fading ( $\varepsilon_h = 0.02$ ,  $K = 256$ ,  $\varepsilon_r = 0.0054$ ).

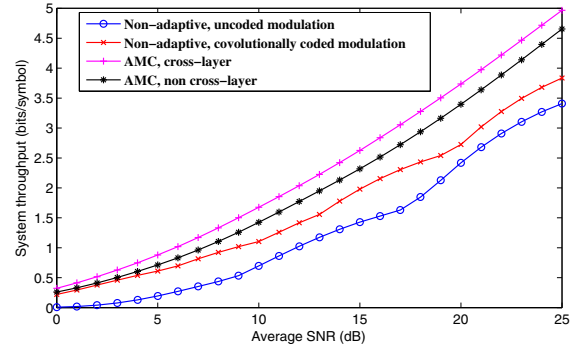


Fig. 6. Throughput performance comparison for slow fading ( $\varepsilon_h = 0.02$ ,  $K = 256$ ,  $\varepsilon_r = 0.0054$ ).

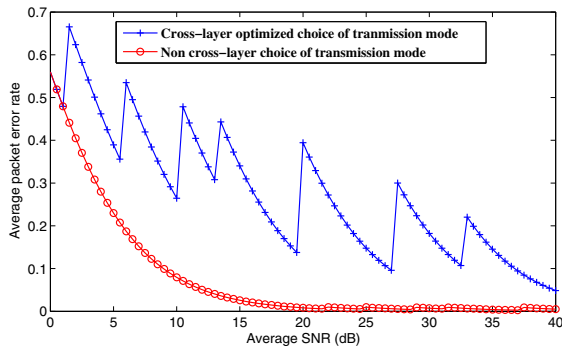


Fig. 5. Average packet error rate in fast Rayleigh fading channels.

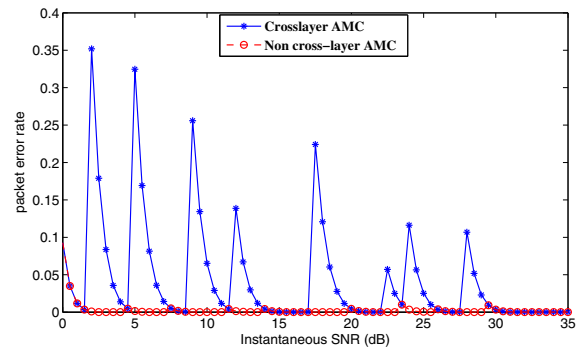


Fig. 7. Packet error rate of traditional and optimal choice of transmission mode at different instantaneous SNR.

$\gamma_n$  given by Eq. (22). For the non-adaptive schemes, the throughput performance is calculated using Eq. (14), where for each given average SNR  $\bar{\gamma}$ , the best performing transmission mode  $n = \arg \max_n \{TP_{slow}(n)\}$  is chosen, where  $TP_{slow}(n)$  is the throughput given by Eq. (14). Note that we have shown two curves of non-adaptive schemes where the candidate transmission modes are limited to either uncoded modulation modes or convolutionally coded modulation modes. It can be seen that both non cross-layer and cross-layer AMC schemes outperform the best performing non-adaptive schemes. The cross-layer AMC design performs better than traditional non cross-layer AMC design across the average SNR range. For example, at an average SNR of 10dB, the cross-layer AMC, traditional AMC, and non AMC design achieve a throughput of 1.68, 1.42, and 1.10 bits/symbol, respectively.

Fig. 7 shows the PER of the transmission mode chosen by the proposed cross-layer scheme and traditional non cross-layer scheme at different instantaneous SNRs for the slow fading scenario. It can be seen that while traditional non cross-layer schemes keep the PER very low (below  $P_{loss} = 0.01$ ), the proposed optimized cross-layer scheme allows much larger PER to be corrected by the application-layer Raptor codes.

## V. SUMMARY

This paper studies the tradeoff between physical layer rate and application-layer Raptor code rate for rateless coded communication systems. We consider both slow and fast

Rayleigh fading for single-user and multicast communications. We propose a cross-layer scheme to optimize the set of available physical-layer modulation and coding pairs. In addition, a cross-layer adaptive modulation and coding design is proposed for the slow fading scenario. The system throughput performance that considers both physical-layer code rate and application-layer erasure code rate is analyzed. Numerical results show that the proposed cross-layer design outperforms traditional non cross-layer design significantly in both slow and fast fading channels.

## REFERENCES

- [1] A. Shokrollahi, "Raptor Codes" *IEEE Trans. Inform. Theory*, Vol. 52, pp. 2551-2567, June 2006.
- [2] M. Luby, T. Gasiba, T. Stockhammer, and M. Watson, "Reliable Multimedia Download Delivery in Cellular Broadcast Network," *IEEE Tran. on Broadcasting*, Vol. 53, No. 1, pp. 235-246, March. 2007.
- [3] T. Courtade and R. Wesel, "A Cross-Layer Perspective on Rateless Coding for Wireless Channels," *IEEE International Conference on Communications*, pp. 1-6, 2009.
- [4] Y. Cao and S.D. Blostein, "Cross-layer Raptor coding for broadcasting over wireless channels with memory," *11th Canadian Workshop on Information Theory*, pp. 130-135, May 2009.
- [5] M.-S. Alouini and A. J. Goldsmith, Adaptive modulation over Nakagami fading channels, *Kluwer J. Wireless Commun.*, Vol. 13, no. 12, pp. 1191-1200, May 2000.
- [6] Q.Liu, S.Zhou, and G.B.Giannakis, "Cross-Layer combining of adaptive Modulation and coding with truncated ARQ over wireless links," *IEEE Transactions on Wireless Communications*, Vol 3, No. 5, pp. 1746-1755, Sept. 2004.