ABSTRACT

Conventionally, antenna arrays apply maximal-ratio combining (MRC) directly to received signals. However, the performance of MRC compensates for its high numerical complexity only for low antenna correlation, i.e., rarely, for space-limited base-stations in actual scenarios, where the azimuth spread (AS) is random and predominantly small. Maximal-ratio eigencombining (MREC), i.e., MRC of the outputs of a partial Karhunen-Loeve Transform (KLT) of the received signals, can attain available diversity gain more efficiently. Herein, MREC is adapted to the AS with a bias-variance tradeoff criterion (BVTC) that selects for KLT dominant eigenvectors of the channel correlation matrix that are computed with a deflation-based projection-approximation subspace tracking (PASTd) algorithm. Optimum and suboptimum channel fading estimation and weight–signal combining are evaluated. Simulations indicate that PASTd-BVTC-based suboptimum MREC with optimum fading estimation can perform similarly to the much more complex optimum-MRC approach.

1. INTRODUCTION

Wireless communications standards (3GPP/2, WiMAX, WiFi, etc.) specify smart antennas [1] designed to take advantage of array and diversity gains [2, Sections 5.2, 5.3]. However, for receive antennas, full array gain is only achievable with coherent combining. Then, full diversity gain is only achievable for uncorrelated fading channel gains. However, channel fading estimation errors [3, Section 3.6] and received-power azimuth spread (AS) [4] can seriously limit these gains for classical antenna array techniques such as diversity combining and statistical beamforming [3,5,6,7,8].

In maximal-ratio combing (MRC) the received signal vector is to be linearly combined with the channel fading gain vector. Estimation of the fading gains at several antennas can require many multiply and add operations, and, thus, significant baseband power consumption [3] [6] [7]. Furthermore, MRC performs best for uncorrelated fading gains [3] [5] [6]. On the other hand, statistical beamforming (BF) combines the received signal vector with the dominant eigenvector of the channel correlation matrix [3] [5] [6]. BF is simpler than MRC, but only yields array gain, which it maximizes for zero AS. However, in typical urban (TU) scenarios, the base station ‘sees’ random but predominantly-small AS [4]. Therefore, in practice BF and MRC may periodically underperform or have oversized numerical complexity [6] [7].

Maximal-ratio eigencombining (MREC) [3] [5] [6] [9] consists of projecting the received signal vector onto dominant eigenvectors of the channel correlation matrix — i.e., the Karhunen-Loeve Transform (KLT) [9] [10] — followed by MRC. Therefore, MREC is a superset of MRC and BF that has been promoted as able to more efficiently and effectively extract the available array and diversity gains, and even to simplify MRC performance analysis [3] [6] [9] [11] [12].

MREC, MRC, and BF performance and complexity have recently been compared for both perfectly and imperfectly known channel eigenstructure (eigenvectors and eigenvalues) and fading factors [5] [6] [7] [9] [11] [12]. Herein, we complement previous work with an evaluation of MREC, MRC, and BF for optimum (exact) vs. suboptimum (approximate) weight–signal combining implementation, and for optimum vs. suboptimum fading estimation [3] [6] for: 1) the 3GPP spatial channel model for base-station receive antenna arrays in typical urban (TU) scenarios [13]; 2) low-complexity, efficient channel eigenstructure tracking [14]; 3) MREC order (i.e., number of KLT eigenvectors) selected adaptively using the bias-variance tradeoff criterion (BVTC) [10].

The paper is organized as follows. Section 2 presents the transmitted signal, channel fading, and AS models. Section 3 describes approximate and exact MREC, and its relationships with MRC and BF, as well as channel fading and eigenstructure estimation methods, along with their numerical complexities. Section 4 shows simulation results for the performance and complexity of BF, MRC, and BVTC-based adaptive MREC.
2. SIGNAL, CHANNEL FADE, AND AS MODELS

2.1. Received Signal Model

A mobile station transmits BPSK signal through a frequency-flat Rayleigh fading channel. At an $L$-element base-station antenna array, the received signal vector after demodulation, matched-filtering, and symbol-rate sampling is [6]

$$\tilde{y} = \sqrt{E_s} \mathbf{b} + \tilde{n}$$  \hspace{1cm} (1)

where $b$ is the equiprobable $\pm 1$ transmitted symbol, and $E_s$ is the energy transmitted per symbol (bit). The fading channel vector, $\mathbf{h}$, and receiver noise, $\tilde{n}$, are assumed mutually uncorrelated zero-mean circularly-symmetric complex Gaussian random vectors [2, p. 39]. The noise vector is assumed temporally and spatially white with $\mathcal{N}_0$ per-element variance, i.e., $\tilde{n} \sim \mathcal{N}(0, \mathbf{I}_N)$. The distribution of the channel gain vector $\mathbf{h}$ is completely described by the channel (Hermitian) correlation matrix, $\mathbf{R}_h \Rightarrow E(\mathbf{h}\mathbf{h}^H)$, whose eigenvalues are real-valued and non-negative, and are hereafter considered ordered as $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L \geq 0$. The corresponding, orthonormal, eigenvectors are denoted as $\mathbf{u}_i, i = 1, \ldots, L \Rightarrow 1 : L$. The eigendecomposition of $\mathbf{R}_h$ is then described by $\mathbf{R}_h = \mathbf{U}_L \mathbf{\Lambda}_L \mathbf{U}_L^H = \sum_{i=1}^{L} \lambda_i \mathbf{u}_i \mathbf{u}_i^H$, where $\mathbf{\Lambda}_L$ and $\mathbf{U}_L$ are a diagonal and a unitary matrix formed with the eigenvalues and eigenvectors of $\mathbf{R}_h$, respectively. Hereafter, the term dominant eigenvectors refers to the set of eigenvectors corresponding to the dominant eigenvalues, i.e., those eigenvalues which contribute most to the trace of $\mathbf{R}_h$.

2.2. Power Azimuth Spectrum (P.A.S.) and Azimuth Spread (AS) Models

In typical urban (TU) environments the received signal power appears dispersed in azimuth angle following a Laplacian distribution [4] [13]. The Laplacian power azimuth spectrum (p.a.s.) is described by [3, p. 136] [5, Eqns. 14, 15] [13, Section 4.5.4] as a function of the mean angle of arrival and of the azimuth spread (AS), which is (approximately) the root second central moment of the p.a.s. [4]. The correlation between two antenna elements can then be computed using the expressions [3, Eqns. (4.3),(4.4), pp. 136-137] [5, Eqn. 16].

Let us consider the ‘TU-32’ scenario described in [4, Table I] for which the base-station AS, measured in degrees, is well-modeled as a lognormally-distributed random variable, i.e., $AS = 10^{0.47 \beta(0.1)+0.74}$ [4, Table II]. Random AS samples yield Pr$(1^\circ < AS < 20^\circ) \approx 0.8$ [6], which implies preponderantly-high (over 0.5) inter-antenna correlation values for compact antenna arrays (with unitary normalized inter-element distance, $d_n = 1$, i.e., physical distance equals half of the carrier wavelength) [6, Fig. 1]. Finally, the received signal experiences AS fluctuation described by the correlation expression $\rho_{AS}(d) = e^{-d/d_{AS}}$ [4, Eqn. (14)], where $d$ is the mobile displacement and $d_{AS} = 70$ m [4, Fig. 4] is the AS decorrelation distance. The AS fluctuation is several orders of magnitude slower than the Doppler-induced fading [6, p. 918].

3. MREC, MRC, AND BF

3.1. MREC Description

We summarize below from [6, Section III.A.1] the steps of maximal-ratio eigencombining (MREC) of order $N = 1 : L$, denoted hereafter as MREC$_N$:

1. The $L \times N$, full-column rank, matrix $\mathbf{U}_N \triangleq [\mathbf{u}_1 \mathbf{u}_2 \ldots \mathbf{u}_N]$ transforms the signal vector from (1) into

$$\mathbf{y} = \sqrt{E_s} \mathbf{b} + \mathbf{n},$$  \hspace{1cm} (2)

where

$$\mathbf{y} \triangleq \mathbf{U}_N^H \tilde{y}, \quad \mathbf{h} \triangleq \mathbf{U}_N^H \mathbf{h}, \quad \mathbf{n} \triangleq \mathbf{U}_N^H \tilde{n}.$$  \hspace{1cm} (3)

This is the well-known Karhunen-Loeve Transform (KLT) [14]. The elements of the N-dimensional vectors $\mathbf{y}$ and $\mathbf{h}$ are hereafter referred to as eigenbranches and eigen-gains, respectively. Our assumptions about the fading and noise imply that $\mathbf{h} \sim \mathcal{N}(0, \mathbf{A}_N)$, i.e., the eigen-gains are independent, with variances $\sigma^2_{\lambda_i} \triangleq E(|h_i|^2) = \lambda_i$, and that $\mathbf{n} \sim \mathcal{N}(0, \mathbf{N}_0 \mathbf{I}_N)$.

2. The transformed signal vector is linearly combined based on the maximal-ratio combining criterion [15], with the transpose complex-conjugate of

$$\mathbf{w}_{\text{MREC}} = \mathbf{h}.$$

Note that MREC$_{N=1}$ represents the conventional maximal average (over fading and noise) signal-to-noise-ratio (SNR) beamforming or statistical beamforming (BF) technique. BF is typically adopted in scenarios with high antenna correlation, to take advantage of array gain [2, Section 5.3]. On the other hand, by skipping Step 1, MREC reduces to the conventional maximal-ratio combining (MRC) technique [15]. MRC is commonly adopted to yield diversity gain [2, Section 5.2] for uncorrelated antennas. Since, as mentioned in Section 2.2, interbranch correlation for compact antennas is predominantly over 0.5 but not always near 1, MREC$_{N>1}$ was recently suggested [3] [6] to cover the BF–MRC performance and complexity middle-ground more effectively.

3.2. Channel Estimation

MRC does not require channel eigenstructure knowledge, but requires estimation of the $L$ channel fading gains. On the other hand, BF requires a single channel eigengain and eigenvector. Finally, MREC$_N$ requires estimates of $N$ channel eigen-gains and the associated eigenstructure. The gains and eigen-gains vary at the Doppler rate, i.e., much faster than the rate of
AS (i.e., eigenstructure) variation. Consequently, the symbol- and channel-rate signal processing (KLT, fading estimation, weight–signal combining) are the important consumers of baseband processing resources (chip area and power) [3, Chapter 5] [7]. Eigenset structure estimation is computationally much less demanding [11].

3.2.1. Channel Eigenstructure Estimation

Previously, we considered only the ideal case of perfectly known channel eigenstructure [3] [6]. For more practical relevance, herein we consider exponentially-weighted sample correlation matrix updating, by using samples of the received signal vector as in [11, Eqn. (13)]. The number of symbols between these samples, denoted hereafter with $K$, is lower-bounded by the maximum normalized Doppler shift [6, Table I] (to ensure temporal decorrelation) and is upper-bounded by the AS decorrelation distance (assuming that no other factors determine antenna correlation fluctuations; therefore, the mean angle of arrival is assumed fixed, at zero). Then, deflation-based projection subspace tracking (PASTd) sequentially computes the required eigenvectors and eigenvalues as shown in [14, Table II], for per-symbol numerical complexity (number of multiplications/ additions with complex numbers) of about $4LN/K$ for MREC$_N$.

3.2.2. SINC and MMSE PSAM Gain/Eigengain Estimation

When the transmitter employs pilot-symbol-aided modulation (PSAM) [3, Section 2.5.1, p. 33] the channel gains or eigengains can be estimated at the receiver by pilot-sample interpolation. This approach is characterized by the PSAM slot length $M_s$ (a slot consists of one pilot symbol followed by $M_s - 1$ data symbols) and by the interpolator size $T$ (the number of pilot samples employed for estimation).

Two PSAM-based fading estimation methods have been proposed in [7, Section 2.6] [3, Section 3.6] [8, Section III.B]: 1) data-independent, suboptimal, entitled SINC PSAM (because the time-response of the interpolation filter approaches a sinc function); 2) data-dependent, optimal, entitled MMSE PSAM (from minimum mean-squared-error). SINC and MMSE PSAM are used for the numerical results shown later, for maximum normalized Doppler shift of $f_m = 0.01$ (i.e., the symbols arrive at a rate 100 higher than the channel fading rate, e.g., when a mobile station with velocity $v = 60$ km/h transmits at rate $f_s = 10$ kbps on a carrier with frequency $f_c = 1.8$ GHz), with $M_s = 7$ and $T = 11$ [3, p. 36].

3.2.3. Exact and Approximate MRC and MREC

Let us assume that the $i$th eigengain, $h_i$, and its estimate, $g_i$, are jointly-Gaussian, and that $\sigma_{h_i g_i} \triangleq E(h_i g_i^\dagger)$, $\sigma_{g_i}^2 \triangleq E(|g_i|^2)$, and the correlation coefficient of $h_i$ and $g_i$, $\rho_i$, are known. Then, the corresponding component of the maximum-likelihood weight vector is [3, Eqn. (3.140), p. 88] [6, Eqn. (23)]

$$[w_{e,N}]_i = \frac{1}{\lambda_i} \frac{\sigma_{h_i g_i}}{\lambda_i (1 - |\mu_i|^2) + 1} g_i, \quad i = 1 : N.$$  (5)

This optimum combining approach has been entitled exact MREC [3] [6]. Note that, for perfect channel knowledge, we have $[w_{e,N}]_i = h_i$, as in (4).

On the other hand, [3, Eqn. (3.143), p. 93] [6, Eqn. (11)]

$$[w_{a,N}]_i = g_i, \quad i = 1 : N,$$  (6)

yields suboptimal performance and has therefore been entitled approximate MREC. (Note that, for perfect channel knowledge, $[w_{a,N}]_i = h_i$.) Compared to the approximate-MREC weights from (6), the exact-MREC weights from (5) require correlation information [3, Table 3.2, p. 83].

Approximate MRC combines the received signal vector $\mathbf{y}$ from (1) with the estimate $\mathbf{g}$ of the channel gain vector $\mathbf{h}$. This approach is simple but suboptimum if knowledge about the channel and noise statistics is available. Given this knowledge, the complicated optimum combining approach, entitled exact MRC, is described in [3, Appendix A].

3.2.4. MREC is a Superset of BF and MRC

MREC$_{N=1}$ represents the classical statistical beamforming (BF) approach [11]. On the other hand, for perfectly known channel eigenstructure, the same linear method (e.g., SINC or MMSE PSAM) for gains and eigengains estimation, and the same weight–signal combining method (i.e., exact or approximate), MREC$_{N=1}$ (full MREC) is performance-equivalent with MRC [3, Section 3.9] [6]. Results not shown here have indicated that for PASTd-based eigenstructure estimation full MREC can still yield near-MRC performance.

3.2.5. Fading Estimation and Weight–Signal Combining Implementation Assumptions

In this paper, some scalars and matrices (mentioned below) that depend on channel statistics, and thus fluctuate very slowly, at the AS rate, are assumed perfectly known. In practice, their estimation would somewhat degrade signal-detection performance and increase processing complexity. Hereafter, we disregard such effects based on the fact that PASTd-based eigenstructure estimation has low impact on the MREC performance and complexity. The simulation results shown later have been obtained under the following assumptions.

For MRC, the channel gain vector is estimated using [3, Eqn. (3.110), p. 83] for SINC PSAM, and using [3, Eqn. (3.113), p. 83] for MMSE PSAM. The channel-statistics-dependent interpolation matrix required for MMSE PSAM [3, Eqn. (3.116), p. 84] is hereafter assumed perfectly known. Further, given an estimate of the channel vector, implementation of exact MRC [3, Appendix A] requires several correlation matrices that are also assumed perfectly known.
It is known as the *bias-variance tradeoff* criterion (BVTC) because (7) balances optimally the loss incurred by removing the weakest \((L - N)\) intended-signal contributions (the first term) against the residual-noise contribution (the second term).

### 4. Simulation Results

Let us assume a BPSK signal transmitted by the mobile station, base-station uniform linear array with \(L = 5\) and \(d_0 = 1\), \(f_m = 0.01\), \(M_S = 7\), and \(T = 11\). At each value of the bit-SNR \(\gamma_b\) — defined to include the pilot-symbol energy (equal to that of the data symbol), as in [3, Eqn. (2.84), p. 38] — the same 30 independent AS samples from the log-normal distribution described in Section 2.2 were considered. The mean and standard deviation of the AS samples is about 10° and 11°, respectively, and \(\Pr(10° < AS < 20°) \approx 0.8\). For each AS sample, the first 120 slots were used for training (i.e., initial channel fading and eigenstructure estimation). Afterwards, the data symbols (6 per slot) of 70000 slots were detected. The channel eigenstructure was estimated with the PASTd algorithm, taking one received signal vector sample every 6 slots apart (i.e., \(K = 42\)), ensuring low temporal correlation of the samples [3, Fig. 2.4, p. 24].

The upper subplot in Fig. 1 shows, vs. the bit-SNR, the average (over noise, fading, and AS) bit error rates (BER) for BF, BVTC MREC, full MREC, and MRC, for SINC PSAM channel estimation and approximate combining. The lower subplot shows the complexity of BVTC MREC divided by that of MRC, as well as the MREC order selected by the BVTC, averaged over AS.

Note first that, since MRC and MREC yield diversity gain, they can greatly outperform BF. Furthermore, by *approximate* combining of only strong eigenbranches, BVTC MREC actually outperforms MRC by about 0.8 dB at BER \(= 10^{-3}\). These and other (not shown) numerical results also indicate that, unlike for MRC, MREC performance reaches an error floor. The lower subplot indicates that, for low bit-SNR, BVTC MREC can be significantly less complex than MRC, whereas, at high bit-SNR, BVTC MREC can be somewhat more complex than MRC. Thus, for SINC PSAM, also a criterion for switching between BVTC MREC and MRC may be required.

Fig. 2 shows results for SINC PSAM and *exact* combining. At non-high SNR, MRC and full-MREC very slightly outperform BVTC MREC, although MRC can be significantly more complex than BVTC MREC because exact combining increases complexity noticeably for MRC but not for MREC. For example, in the shown bit-SNR range, BVTC MREC can reduce complexity vs. MRC by 15% to 75%. However, due to the MREC error floor it may again be necessary to switch from BVTC MREC to MRC at high bit-SNR.

Figs. 1 and 2 reveal a small performance gain (about 1.3 dB at BER \(= 10^{-3}\)) of exact MRC over approximate MRC, which may not warrant the 42% complexity increase (see Table 1).

### Table 1. Percentage Relative Change in Numerical Complexities for MRC, full-MREC

<table>
<thead>
<tr>
<th>column</th>
<th>Approx, SINC</th>
<th>Approx, MMSE</th>
<th>Exact, SINC</th>
<th>Exact, MMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SINC</td>
<td>0, 0</td>
<td>-367, 0</td>
<td>-42, -6</td>
<td>-408, -6</td>
</tr>
<tr>
<td>MMSE</td>
<td>78, 0</td>
<td>0, 0</td>
<td>70, -6</td>
<td>-9, -6</td>
</tr>
<tr>
<td>Exact</td>
<td>29, 5</td>
<td>-229, 5</td>
<td>0, 0</td>
<td>-259, 0</td>
</tr>
<tr>
<td>SINC</td>
<td>80, 5</td>
<td>8, 5</td>
<td>72, 0</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

For MREC, separate estimation of the channel eigengains is permitted due to their statistical independence. However, unlike SINC PSAM, MMSE PSAM requires channel-statistics-dependant interpolation vectors [3, Table 3.1, p. 83] [3, Table 3.7, p. 132], which are herein assumed perfectly known. Furthermore, the correlations required to compute the weights for exact eigencombining from those for approximate combining — see (5) and (6) — are also assumed perfectly known.

### 3.3. Numerical Complexity

Hereafter, we look at the per-symbol number of complex-valued multiplications/additions incurred by KLT, fading gain and eigengain estimation, and weight-signal combining, in MRC and MREC. Table 1 shows the percentage relative change in numerical complexity for MRC and full-MREC (MRC,full-MREC), for \(L = 5\) and \(T = 11\), when switching from the estimation and combining methods shown in the leftmost column to the methods shown in the upper row. The formula used is \(\left|\frac{\text{column} - \text{row}}{\text{column}}\right| \cdot 100\), where *column* and *row* represent the numerical complexities computed using [6, Table II] [3, Table 3.7, p. 132] plus (for MREC) the PASTd contribution. A minus sign indicates complexity increase. Note that MRC complexity increases several-fold when switching from SINC to MMSE PSAM. MRC complexity also increases significantly for SINC PSAM when switching from approximate to exact combining. For MMSE PSAM, channel gain estimation is so complex that switching from approximate to exact combining does not incur a significant relative complexity increase. MREC exhibits no change when switching from SINC to MMSE PSAM, and only small complexity increases when switching from approximate to exact combining.

### 3.4. Optimum Order Selection for MREC

The following simple criterion proposed in [10] was previously found effective for MREC adaptation to the AS, for perfectly known eigenstructure [6] [7]:

$$
\min_{N=1:L} \left[ E_s \cdot \sum_{i=N+1}^{L} \lambda_i + N_0 \cdot N \right].
$$

(7)
On the other hand, approximate and exact BVTC MREC have indistinguishable performance and similar complexity, and perform only about 0.5 dB worse than exact-MRC, which can be much more complex. Thus, for SINC PSAM, it may suffice to deploy low-complexity approximate BVTC MREC to achieve near-optimum performance.

Results for MMSE PSAM are shown for approximate combining only, in Fig. 3, because no performance improvement was noticed with the higher-complexity exact combining approach. BVTC MREC now performs only about 0.7 dB worse than MRC at BER $= 10^{-3}$, for a fraction of the MRC complexity. In the shown bit-SNR range, BVTC MREC can reduce complexity by about 75% to 90% over MRC. For exact combining the complexity reduction with BVTC MREC vs. MRC is even more appealing.

Figs. 1 and 3 show that MREC incurs error floor for SINC PSAM but not for MMSE PSAM. Note, however, that although MRC also employs fading estimation, it does not display an error floor. Thus, the MREC error floor for SINC PSAM is due to compounded SINC PSAM channel fading estimation error and PASTd eigenstructure estimation error.

Comparing Figs. 1 and 3 reveals that, at $BER = 10^{-3}$, MMSE PSAM outperforms SINC PSAM by about 1.7 dB and 3 dB for BVTC MREC and MRC, respectively. For BVTC MREC, the high-bit-SNR gain with MMSE PSAM over SINC PSAM can be even larger since MMSE PSAM exhibits no error floor. The lack of error floor for MMSE PSAM-based MREC also eliminates the need for switching from BVTC MREC to MRC at high bit-SNR. Furthermore, Table 1 indicates that the MMSE vs. SINC PSAM performance gain is obtainable at no additional complexity with BVTC MREC, but for very significant additional complexity with MRC. Thus, for MMSE PSAM, approximate BVTC MREC can yield near-exact-MRC performance for the low complexity of SINC-PSAM-based BVTC MREC.

Finally, similar results (not shown) have been obtained for variable AS, with distance (or, equivalently, temporal) correlation characterized by the realistic expression included at the end of Section 2.2.

### 5. SUMMARY AND CONCLUSIONS

The paper has evaluated the performance and complexity of statistical beamforming (BF) and maximal-ratio combining (MRC), and of their superset known as maximal-ratio eigendecomposition (MREC). A low-complexity, efficient projection-approximation subspace tracking algorithm has been employed for updating the channel eigenvalues and eigenvectors, using samples of the received signal vector. The bias-variance tradeoff criterion (BVTC) has been employed for MREC adaptation to the actual spatial correlation. It has emerged that a suboptimum (approximate) implementation of BVTC MREC yields similar performance and complexity as the optimum (exact) implementation, and nearly the same performance as the actual optimum, but very complex, exact-MRC approach. Furthermore, unlike MRC, BVTC MREC can approach the
Fig. 3. Top: Average (over noise, fading, and AS) error rate vs. bit-SNR, for BF, BVTC MREC, full MREC, and MRC, for MMSE PSAM and approximate combining. Bottom: Numerical complexity of BVTC MREC relative to that of MRC, and BVTC output, averaged over the AS.

6. REFERENCES


