Linear dispersion for single-carrier communications in frequency selective channels

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***Abstract*— Linear dispersion coded orthogonal frequency division multiplexing (LDC-OFDM) has recently been proposed to improve joint frequency and time diversity. This paper investigates whether LDC are able to support joint frequency and time diversity for single-carrier block communications in time-varying frequency selective fading channels, and proposes linear dispersion coded cyclic-prefix single-carrier modulation (LDC-CP-SCM), which utilizes LDC across multiple CP-SCM blocks. LDC-CP-SCM uses a layered two-stage LDC decoding strategy, and is thus backwards-compatible to CP-SCM systems. This paper analyzes the diversity properties of LDC-CP-SCM, and provides a sufficient condition for LDC-CP-SCM to maximize all available joint frequency and time diversity gain and coding gain. For the LDC considered, simulations show that with and without carrier frequency offset (CFO) effects, LDC-CP-SCM may outperform both CP-SCM and LDC-CP-OFDM in time-varying frequency selective channels. This paper also shows that LDC-CP-SCM with forward error correction (FEC) may outperform CP-SCM with FEC over time.**

***Index Terms*— single carrier modulation, frequency domain equalization, linear dispersion codes, OFDM, diversity order, cyclic-prefix, peak-to-average power ratio, carrier frequency offsets, forward error correction, MMSE, low complexity, frequency selective channels**

I. Introduction

Recently, as an important candidate for broadband communications, cyclic-prefix single-carrier modulation (CP-SCM) with frequency-domain equalization (FDE) has attracted a lot of attention. Similar to orthogonal frequency-division multiplexing (OFDM), CP-SCM FDE provides much lower computational complexity than conventional time-domain equalization techniques, especially for long impulse response tail channels [1]. Unlike OFDM, CP-SCM does not suffer high peak-to-average power ratio (PAPR) as well as sensitivity to carrier-frequency offsets (CFO) [2] and nonlinear distortions [3]. CP-SCM FDE has been recommended for fixed wireless broadband standard IEEE 802.16 [**?**].

Linear dispersion codes (LDC), as pioneered by Hassibi and Hochwald for space time coding over block flat fading channels [4], possess high coding rates of up to one [4]. Recently, high rate LDC have been employed to obtain joint frequency and time diversity in OFDM, known as LDC-OFDM [5]. Although LDC has been applied to multicarrier communications, no proposal has been found in literature to apply high-rate LDC in single stream single carrier communications. This paper investigates the application of LDC to achieve joint frequency-time diversity in CP-SCM for frequency selective channels.

This paper proposes linear dispersion coded cyclic-prefix single-carrier modulation (LDC-CP-SCM), which applies linear dispersion codes across multiple CP-SCM blocks. Unlike LDC-OFDM, in which LDC exploits both frequency and time diversity available in the channels, simulations reveal that LDC primarily improves time diversity in LDC-CP-SCM and significantly outperforms both CP-SCM and LDC-CP-OFDM in time-varying frequency selective channels.

The following notation is used: $(\cdot)^{\dagger}$ denotes matrix pseudoinverse, $[\cdot]^{T}$ matrix transpose, $[\cdot]^{\mathcal{H}}$ matrix transpose conjugate, $E[\cdot]$ expectation, j is the square root of -1, $\mathbf{I}_K$ denotes identity matrix of size $K \times K$, $\mathbf{0}_{m\times n}$ denotes a zero matrix of size $m \times n$, $C^{m\times n}$ denotes a complex matrix with dimensions $m \times n$, $[\mathbf{A}]_{a,b}$ denote the (a, b) entry of matrix $\mathbf{A}$, and $diag(\cdot)$ transforms the argument from a vector to a diagonal matrix, matrix $\mathbf{F}_M$ of size $M \times M$ denotes the discrete Fourier transform (DFT) matrix representing the M-point FFT with elements

$$[\mathbf{F}_M]_{a,b} = \left(1/\sqrt{M}\right) \exp\left(-j2\pi(a-1)(b-1)/M\right).$$

II. LDC Encoding

Assume that an uncorrelated data sequence has been modulated using complex-valued source data symbols chosen from an arbitrary, e.g. r-PSK or r-QAM, constellation. A $T \times M$ LDC matrix codeword, $\mathbf{S}_{LDC}$, is transmitted over M channels and occupies T channel uses and encodes Q source data symbols. Denote the LDC codeword matrix as $\mathbf{S}_{LDC} \in C^{T\times M}$, and $\mathbf{A}_q \in C^{T\times M}, \mathbf{B}_q \in C^{T\times M}, q = 1, ..., Q$ are called dispersion matrices. Unlike [4], which considers LDC only as space time codes, we consider LDC as a general framework of complex matrix codes.

Just as in [5], we consider the case $\mathbf{A}_q = \mathbf{B}_q, q = 1, ..., Q$. We have LDC encoding in the matrix form equation,

$$vec(\mathbf{S}_{LDC}) = \mathbf{G}_{LDC}\mathbf{s}, \tag{1}$$

where $\mathbf{s} = [\ s_1 \ \ \cdots \ \ s_Q\ ]^T$ is the source complex symbol vector,

$$\mathbf{G}_{LDC} = [vec(\mathbf{A}_1), ..., vec(\mathbf{A}_Q)] \tag{2}$$

is the LDC encoding matrix. Details can be found in [5]. To estimate the data symbol vector in (1), we may calculate the Moore-Penrose pseudo-inverse of LDC encoding matrix G_{LDC} offline and store the result.

III. Single-carrier communications model

Assume the communications channel experiences frequency-selective fading, and the channel for the k-th SCM block is modeled as an L th-order FIR filter with impulse response $\mathbf{h}^{(k)} = \left[h_0^{(k)}, ..., h_L^{(k)}\right]^T$. Channel coefficients are constant within one SCM block but change statistically independently across different SCM blocks. Each SCM block is of size $P = N_C + N_g$, including a data symbol block of size N_C and a guard interval of size $N_g \geq L$ to avoid inter-block interference.

Denote $\mathbf{x}_{SC}^{(k)}$ as the channel data symbol vector transmitted during the k-th SCM block of size $N_C \times 1$, and $\mathbf{x}_{SC}^{(k)} = \left[x_{SC(1)}^{(k)}, ..., x_{SC(N_C)}^{(k)}\right]^{\mathcal{T}}$, where $x_{SC(p)}^{(k)}, p = 1, ..., N_C$ is the p-th data symbol of the k-th SCM block in sequence. Each receive antenna experiences additive white complex Gaussian noise. The system signal-to-noise-ratio (SNR) is denoted by ρ.

Before transmission, a cyclic prefix (CP) guard interval is appended to each CP-SCM block. The CP is then removed at the receiver. The effective channel of the k-th SCM block is a circulant matrix $\mathbf{H}_{CP\_SC}^{(k)}$ with elements $\left[\mathbf{H}_{CP\_SC}^{(k)}\right]_{a,b} = h_{((a-b) \bmod N_C)}^{(k)}$. Hence, the CP-SCM block system can be modeled as

$$\mathbf{r}_{CP\_SC}^{(k)} = \sqrt{\rho}\mathbf{H}_{CP\_SC}^{(k)}\mathbf{x}_{SC}^{(k)} + \mathbf{v}_{CP\_SC}^{(k)} \tag{3}$$

where $\mathbf{r}_{CP\_SC}^{(k)}$ is the received block after CP removal, and $\mathbf{v}_{CP\_SC}^{(k)}$ is the corresponding noise vector.

At the receiver, the received block $\mathbf{r}_{CP\_SC}^{(k)}$ is first processed by an FFT to generate block $\mathbf{y}_{CP\_SC}^{(k)} = \mathbf{F}_{N_C}\mathbf{r}_{CP\_SC}^{(k)}$.

Due to the circulant property of $\mathbf{H}_{CP\_SC}^{(k)}$, can be decomposed as

$$\mathbf{H}_{CP\_SC}^{(k)} = \left[\mathbf{F}_{N_C}\right]^H \mathbf{D}_{CP\_SC}^{(k)}\mathbf{F}_{N_C},$$

where $\mathbf{D}_{CP\_SC}^{(k)}$ is diagonal with

$$\left[\mathbf{D}_{CP\_SC}^{(k)}\right]_{pp} = \sum_{l=0}^{L} h_l^{(k)} \exp\left(-j2\pi l(p-1)/N_C\right).$$

Thus, the frequency domain system equation is

$$\mathbf{y}_{CP\_SC}^{(k)} = \sqrt{\rho}\mathbf{D}_{CP\_SC}^{(k)}\mathbf{F}_{N_C}\mathbf{x}_{SC}^{(k)} + \mathbf{F}_{N_C}\mathbf{v}_{CP\_SC}^{(k)}. \tag{4}$$

IV. PROPOSED LDC BASED SINGLE-CARRIER BLOCK COMMUNICATIONS

A. Proposed system structure

Similar to [5], we adopt a layered approach that utilizes a two-step-estimation (TSE) procedure -

1) *Signal estimation per channel use:*
Signals in each channel use are estimated. No immediate signal detection is performed. (Channel knowledge for each channel use is required for each estimate. Over different channel uses, channel matrices may vary.)
2) *Data symbol estimation and detection per LDC codeword:*
After signal estimation for T channel uses corresponding to one LDC block is completed, source data symbols are estimated from estimated LDC-encoded symbols. (In this step, channel knowledge is not required). Bit detection is then performed.

Note that the TSE approach requires that LDC coded symbols are uncorrelated either per matrix element or per SCM block, otherwise the system equation (4) may be invalid.

One LDC-CP-SCM block consists of T adjacent SCM blocks. In addition, one LDC-CP-SCM block includes D LDC codewords, each of size $T \times N_{F(i)}, i = 1, ..., D$, where $N_{F(i)}$ is the number of channel symbols within one SCM block, which the i-th LDC codeword is across. Thus, the maximal size of one LDC-SCM block is $T \times N_C$.

One LDC-SCM block is the transmitted during the period of T SCM blocks, a guard interval (CP) is added to each SCM block before transmission. After the transmitted channel symbols are corrupted in the channels, the receiver removes the guard interval and performs equalization.

For the LDC-CP-SCM receiver, frequency-domain equalization (FDE) can be applied as shown in Figure 1.

B. LDC-CP-SCM receiver

Denote the LDC encoding matrix of the i-th LDC matrix codeword $\mathbf{S}_{LDC}^{(i)} \in C^{T \times N_{F(i)}}$ as $\mathbf{G}_{LDC}^{(i)}$, which encodes source data symbol vector with zero mean, unit variance, $\mathbf{s}^{(i)} = \begin{bmatrix} s_1^{(i)} & s_1^{(i)} & \cdots & s_{Q_i}^{(i)} \end{bmatrix}^{\mathcal{T}}$ into $vec(\mathbf{S}_{LDC}^{(i)})$, where Q_i is the number of source data symbols in $\mathbf{s}^{(i)}$, $i = 1, ..., D$.

1) First estimation step - SCM demodulation: In the proposed algorithm, LDC decoding is independent of SCM signal estimation. In this way, the proposed system is backwards-compatible to conventional SCM systems. A significant advantage arising from LDC-CP-SCM decoding is that channel coefficients need not remain constant over multiple SCM blocks.

In Section VI, performance is investigated using MMSE equalization. Assuming that single-carrier symbols are normalized to unit variance, the respective frequency and time domain equalizers are given by [6]

$$\begin{aligned}\mathbf{G}_{CP\_SC}^{(k)} = \sqrt{\rho}\mathbf{C}_{\mathbf{x}_{SC}^{(k)}}\left(\mathbf{D}_{CP-SC}^{(k)}\right)^{\mathcal{H}} \cdot \\ \left(\mathbf{I}_{N_C} + \rho\mathbf{D}_{CP-SC}^{(k)}\mathbf{C}_{\mathbf{x}_{SC}^{(k)}}\left(\mathbf{D}_{CP-SC}^{(k)}\right)^{\mathcal{H}}\right)^{-1}\end{aligned} \tag{5}$$

and

$$\widehat{\mathbf{x}_{SC}^{(k)}} = \left[\mathbf{F}_{N_C}\right]^{\mathcal{H}} \mathbf{G}_{CP\_SC}^{(k)}\mathbf{y}_{CP\_SC}^{(k)}. \tag{6}$$

2) Second estimation step - LDC-SCM block decoding: Reorganizing the results of the first estimation step into D estimated LDC matrix codewords, $\widehat{\mathbf{S}_{LDC}^{(i)}}, i = 1, ..., D$, the estimated data symbol vectors corresponding to D LDC blocks are

$$\widehat{\mathbf{s}^{(i)}} = \left[\mathbf{G}_{LDC}^{(i)}\right]^{\dagger} vec(\widehat{\mathbf{S}_{LDC}^{(i)}}) \tag{7}$$

C. Peak-to-average power ratio

Single-carrier complex matrix codes (SCCMC) are currently proposed as space time block codes in the literature, and usually possess lower PAPR than OFDM. However, the PAPR of SCCMC is often higher than that of conventional constellation-based SCM. Fortunately, designing lower PAPR SCCMC is easier than designing lower PAPR OFDM based systems. Some initial efforts in addressing this issue can be found in [7], [8].

D. Carrier frequency offsets

Conventional constellation-based SCM has fewer problems with regard to carrier frequency offsets (CFO). However, LDC are a class of transformations over both phase and amplitude, and CFO effects are due to phase shift sensitivity. We are interested in investigating performance of LDC-CP-SCM under CFO effects. With minor modification, we extend CP-OFDM CFO system model in [9] to CP-SCM as follows:

$$\begin{aligned}\mathbf{y}_{CP\_SC}^{(k)} = \sqrt{\rho}\phi(a)\mathbf{U}_{CFO}\mathbf{D}_{CP\_SC}^{(k)}\mathbf{F}_{N_C}\mathbf{x}_{SC}^{(k)} \\ +\mathbf{F}_{N_C}\mathbf{v}_{CP\_SC}^{(k)}\end{aligned} \tag{8}$$

where

1) $\varepsilon = \Delta f T_s N_C$ is normalized CFO, Δf is CFO and T_s is the channel symbol period;
2) $$\phi(a) = \exp(j2\pi\varepsilon((a-1)P + N_g)/N_C)$$
   and
   $$a = \begin{cases} k(\text{mod} N_{FS}) \text{ if } k(\text{mod} N_{FS}) \neq 0; \\ N_{FS} \text{ if } k(\text{mod} N_{FS}) = 0; \end{cases}$$
   where N_{FS} stands for frequency synchronization rate (FSR) per SCM block. In other words, $(a-1)$ is set to zero every N_{FS} SCM blocks;
3) $\mathbf{U}_{CFO} = \mathbf{F}_{N_C}\mathbf{D}_{CFO}\left[\mathbf{F}_{N_C}\right]^{\mathcal{H}}$, where
   $$\mathbf{D}_{CFO} = \\ diag(\exp(j2\pi\varepsilon(1/N_C)), ..., \exp(j2\pi\varepsilon(N_C/N_C))).$$

For comparison purposes, we also consider CP-OFDM under CFO effects, i.e.,

$$\mathbf{y}^{(k)}_{CP\_OFDM} = \sqrt{\rho}\phi(a)\mathbf{U}_{CFO}\mathbf{D}^{(k)}_{CP\_OFDM}\mathbf{x}^{(k)}_{OFDM} \\ +\mathbf{F}_{N_C}\mathbf{v}^{(k)}_{CP\_OFDM} \tag{9}$$

Based on models (8) and (9), a CFO effect comparison through simulations is provided in Section VI-E.

V. Diversity properties

Since it is easier to consider frequency-domain signals in order to study both temporal and frequency diversity, the chosen object to be analyzed is $\mathbf{z}^{(k)}_{CP-SC} = \mathbf{F}_{Nc}\mathbf{x}^{(k)}_{SC}, k = 1, ..., T$. Thus the whole LDC-CP-SCM block with FFT outer processing in each SCM block can expressed as matrix $\mathbf{C}$

$$\mathbf{C} = \begin{bmatrix} c_1^{(1)} & c_2^{(1)} & \cdots & c_{N_C}^{(1)} \\ c_2^{(2)} & c_2^{(2)} & \cdots & c_{N_C}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{(T)} & c_2^{(T)} & \cdots & c_{N_C}^{(T)} \end{bmatrix},$$

where $c_p^{(k)} = \left[\mathbf{z}^{(k)}_{CP-SC}\right]_{p,1}, p = 1, ..., N_C, k = 1, ...T.$

We write the system equation for the block $\mathbf{C}$ as

$$\mathbf{R} = \sqrt{\rho}\mathbf{M}\mathbf{H} + \mathbf{V}, \tag{10}$$

where receive signal vector $\mathbf{R}$ and noise vector $\mathbf{V}$ are of size $N_C T \times 1$. The chosen frequency symbol diagonal matrix $\mathbf{M}$ is of size $N_C T \times N_C T$, and

$$\mathbf{M} = \\ diag(c_1^{(1)}, ..., c_{N_C}^{(1)}, ..., c_1^{(T)}, ..., c_{N_C}^{(T)}) \ .$$

The channel vector $\mathbf{H}$ is of size $N_C T \times 1$, and

$$\mathbf{H} = \begin{bmatrix} H_1^{(1)}, H_2^{(1)}, ..., H_{N_C}^{(1)}, \\ ..., H_2^{(T)}, H_2^{(T)}, ..., H_{N_C}^{(T)} \end{bmatrix}^{\mathcal{T}},$$

where $H_p^{(k)}$ is the p-th subchannel gain of k-th SCM block in $\mathbf{C}$ in the frequency domain. Thus $H_p^{(k)} = \left[\mathbf{w}_p\right]^T \mathbf{h}^{(k)}$, where

$$\mathbf{w}_p = \left[1, \omega^{p-1}, \omega^{2(p-1)}, \cdots, \omega^{L(p-1)}\right]^{\mathcal{T}}$$

and

$$\omega = e^{-j(2\pi/N_c)}.$$

Consider a pair of matrices $\mathbf{M}$ and $\tilde{\mathbf{M}}$ corresponding to two different blocks $\mathbf{C}$ and $\tilde{\mathbf{C}}$. Then the upper bound pairwise error probability between $\mathbf{M}$ and $\tilde{\mathbf{M}}$ is [10]

$$P\left(\mathbf{M} \to \tilde{\mathbf{M}}\right) \leq \\ \begin{pmatrix} 2r-1 \\ r \end{pmatrix} \left(\prod_{a=1}^{r} \gamma_a\right)^{-1} (\rho)^{-r} \ , \tag{11}$$

where r is the rank of $\mathbf{\Lambda} = \left(\mathbf{M} - \tilde{\mathbf{M}}\right)\mathbf{R_H}\left(\mathbf{M} - \tilde{\mathbf{M}}\right)^{\mathcal{H}}$ and $\mathbf{R_H} = E\left\{\mathbf{H}\left[\mathbf{H}\right]^{\mathcal{H}}\right\}$ is the correlation matrix of $\mathbf{H}$, $\gamma_a, a = 1, \cdots, r$ are the non-zero eigenvalues of $\mathbf{\Lambda}$.

Then the corresponding rank and product criteria are as follows:

1) Rank criterion: The minimum rank of $\mathbf{\Lambda}$ over all pairs of different frequency domain symbol matrices $\mathbf{M}$ and $\tilde{\mathbf{M}}$ should be as large as possible.
2) Product criterion: The minimum value of the product $\prod_{a=1}^{r} \gamma_a$ over all pairs of different frequency domain symbol matrices $\mathbf{M}$ and $\tilde{\mathbf{M}}$ should be maximized.

We remark that we can obtain a sufficient condition for LDC-CP-SCM to achieve full available joint frequency and time diversity in the channels, which is provided in

Theorem 1: In a LDC-CP-SCM system, the rank of frequency domain matrix meets $rank\left(\mathbf{M} - \tilde{\mathbf{M}}\right) = N_C T$.

1) The LDC-CP-SCM system achieves full available diversity order in the frequency selective channels, i.e. $rank\left(\mathbf{\Lambda}\right) = rank\left(\mathbf{R_H}\right)$
2) The sufficient and necessary condition to ensure $rank\left(\mathbf{M} - \tilde{\mathbf{M}}\right) = N_C T$ is
   $$\left[\mathbf{F}_{Nc}\left(\mathbf{x}^{(k)}_{SC} - \tilde{\mathbf{x}}^{(k)}_{SC}\right)\right]_{p,1} \neq 0, k = 1, ..., T, p = 1, ..., N_C$$
3) The corresponding product design criterion for LDC-CP-SCM block is that the minimum of products
   $$\Delta = \prod_{k=1}^{T}\prod_{p=1}^{N_C}\left|\left[\mathbf{F}_{Nc}\mathbf{x}^{(k)}_{SC}\right]_{p,1} - \left[\mathbf{F}_{Nc}\tilde{\mathbf{x}}^{(k)}_{SC}\right]_{p,1}\right|^2 \tag{12}$$
   taken over all pairs of distinct frequency domain symbol matrices $\mathbf{M}$ and $\tilde{\mathbf{M}}$ must be maximized.

The proof is omitted due to space limitations [11]. Note that single-carrier systems are inherently able to achieve some frequency diversity. However, full frequency diversity order is not guaranteed in uncoded single-carrier systems, especially in uncoded CP-SCM systems. Further, the frequency coding gain still could be improved through careful design [12], [13]. A LDC-SCM block consists of multiple SCM blocks, and the LDC-SCM system has potential to achieve joint frequency-time diversity order up to $T(L+1)$. The design strategy of LDC-SCM systems to support a certain order of frequency diversity is different from that of LDC-OFDM, since the code in SCM cannot be designed in parallel across frequency subchannels unless the inverse FFT is used as an outer precoding (observe (4)). It is clear that if IFFT is used as a part of the precoding process for every SCM block at the transmitter, the designed system is no longer a single carrier system, and actually becomes a LDC-OFDM system.

VI. Performance

A. Simulation setup

Perfect channel knowledge is assumed at the receiver but not at the transmitter. The number of data symbols per SCM block, N_C, is 32. All LDC codewords are encoded using Eq. (31) of [4].

The LDC symbol coding rates of the proposed systems used in simulations are all unity. Compared with uncoded CP-SCM systems, no bandwidth is lost unless forward error control is used. The sizes of all LDC codewords are identically $T \times N_F$. An evenly spaced mapping either from LDC to channel data symbol index for LDC-CP-SCM or from LDC to subcarrier index for LDC-CP-OFDM is used in simulations.

The frequency selective channel has $(L+1)$ paths exhibiting an exponential power delay profile, and the guard interval size of each SCM block is set to $N_g = L$. Source data symbols use QPSK modulation in all simulations. For simplicity, the channel is assumed to be constant over a certain number of SCM blocks, and i.i.d. between these groups of blocks. We term this interval as the channel change interval (CCI).

B. Comparison between LDC-CP-SCM and CP-SCM systems

Figures 2 shows the diversity performance comparison of bit error rate (BER) vs. SNR between LDC-CP-SCM and CP-SCM.

When CCI is a multiple of T, i.e. $CCI = 16$, the effects of time diversity in the channels are removed, and it can be observed that the performances of LDC-CP-SCM and CP-SCM are quite similar, which suggests that the LDC-CP-SCM systems using the chosen LDC does not provide notable frequency coding improvement over CP-SCM systems. Note that the chosen LDC is designed for space time block fading channels, which may not be optimal for SCM in frequency selective time varying channels. To obtain frequency diversity improvement, new LDC designs are needed. It is not an easy task to design LDC meeting the design criterion in Theorem 1 as well as maintaining desired lower PAPR as in convectional CP-SCM systems, since the new design should consider the pairwise differences of Fourier transformed coded data symbols instead of coded data symbols themselves, which differs from current existing single-carrier complex matrix code designs.

When CCI is not a multiple of T, i.e. $CCI < 16$, clearly, BER performance of LDC-CP-SCM is remarkably better than that of both uncoded CP-SCM, which is primarily attributed to time diversity. Time diversity order is maximized only if the channel provides block-wise temporal independence. As shown in Figures 2, the performance of LDC-CP-SCM systems is significantly influenced by channel dynamics, i.e., time correlation. At high SNRs, the faster the channel changes, the better the performance. This indicates that LDC-CP-SCM effectively exploits available temporal diversity. In the future, testing on a more accurate model of channel dynamics is needed to obtain a more accurate assessment.

C. Comparison between LDC-CP-SCM and CP-SCM systems using forward error correction

Figure 3 shows a diversity performance comparison of BER vs. SNR between LDC-CP-SCM and CP-SCM using forward error correction (FEC). For low latency, Reed Solomon (RS) codes are chosen. In Figure 3, $RS(a, b, c)$ denotes RS codes with a coded RS symbols, b information RS symbols, and c bits per symbol. In simulations, shortened RS codes are chosen. For fairness of comparison, in CP-SCM systems, we apply RS codes across the same number of CP-SCM blocks as that in LDC-CP-SCM systems, and each RS symbol is distributed within one CP-SCM block. In this way, RS codes are able to improve time and frequency diversity in CP-SCM systems. In LDC-CP-SCM systems, we partition RS(a,b,c) codewords into $\frac{N_C}{N_F}$ groups, and each group of RS symbols are encoded in one LDC codeword within one LDC-CP-SCM block.

It is clear that when using the same RS codes in medium to high SNRs, LDC-CP-SCM with FEC notably outperforms CP-SCM with FEC. For instance, at BER of 10^{-3}, using $RS(16, 12, 8)$, the LDC-CP-OFDM with FEC outperform CP-OFDM with FEC by 2.4 dB. Also note that while CP-SCM with FEC may outperform LDC-CP-SCM without FEC, the data rate of the corresponding CP-SCM with FEC is lower than that of the LDC-CP-SCM without FEC.

D. Comparison between LDC-CP-SCM and LDC-CP-OFDM systems

Figure 4 shows a diversity performance comparison of BER vs. SNR between LDC-CP-SCM and LDC-CP-OFDM. In medium to high SNRs, especially when CCI is not a multiple of T, i.e. $CCI < 8$, based on the chosen LDC, the LDC-CP-SCM significantly outperforms LDC-CP-OFDM systems, which suggests that this LDC-CP-SCM design has larger diversity gain over the tested LDC-CP-OFDM in these channels, and the tested LDC-CP-OFDM is not full joint frequency and time diversity design.

E. Comparison of cyclic-prefix (CP) based systems under CFO effects

In Figure 5, the detrimental effects of CFO are observed. Under the normalized CFO setting of $\varepsilon = 0.02$, CP based systems without CFO outperform those with CFO, especially at higher SNRs. In higher SNRs, the performance loss of LDC-CP based systems due to CFO effects is higher than that of uncoded CP based systems. Although having the highest performance loss under CFO effects, the tested LDC-CP-SCM has the best performance in time varying frequency selective channels.

VII. Conclusions

This paper proposes the use of high-rate LDC in cyclic-prefix single-carrier communications systems in time-varying frequency selective channels. While performance improvement has been previously obtained for multicarrier systems [5], performance of single-carrier systems may also be improved using high-rate LDC, as is shown in this paper. In the LDC-CP-SCM design tested, an increase in time diversity resulted in a performance improvement. However, it may be possible for LDC to improve joint frequency and time diversity, which is a subject of future investigation. This paper provides a sufficient condition for LDC-CP-SCM to maximize all available joint frequency and time diversity gain and coding gain. Simulations reveal that, based on the chosen LDC, LDC-CP-SCM may outperform both CP-SCM and LDC-CP-OFDM in time-varying frequency selective channels, even under CFO effects. This paper also shows that LDC-CP-SCM with forward error correction may outperform CP-SCM with forward error correction over time.

REFERENCES

[1] D.Falconer, S.L.Ariyavisitakul, A.Benyamin-Seeyar, and B.Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun.Mag.*, pp. 58–66, Apr. 2002.

[2] J. Heiskala and J. Terry, *OFDM wireless LANs: a theoretical and practical guide*. SAMS, 2002.

[3] T.Buzid, S.Reinhardt, and M.Huemer, "A comparision of OFDM and non-linear SC/FDE signals: Non-linear SC/FDE signals: non-linear amplification," in *Proc. European Wireless Conf. 2005*, Apr. 2005.

[4] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans.Inform.Theory*, vol. 48, no. 7, pp. 1804–1824, July 2002.

[5] J.Wu and S.Blostein, "Linear dispersion over time and frequency," in *Proc. IEEE ICC 2004*, June 2004.

[6] H. Poor, *An introduction to signal detection and estimation*. Springer Verlag, Mar..1994.

[7] P.Dayal and M.K.Varanasi, "Maximal diversity algebraic space-time codes with low peak-to-mean power ratio," *IEEE Trans.Inform.Theory*, vol. 51, no. 5, pp. 1691–1708, May 2005.

[8] M.O.Damen, H. Gamal, and N.C.Beaulieu, "Linear threaded algebraic space-time constellation," *IEEE Trans.Inform.Theory*, pp. 2372–2388, Oct. 2003.

[9] U.Tureli, P.J.Honan, and H. Liu, "Low-complexity nonlinear least squares carrier offset estimator for OFDM: identifiability, diversity and performance," *IEEE Trans.on Sig.Proc.*, vol. 52, no. 9, pp. 2441–2452, Sept. 2004.

[10] S.Siwamogsatham, M.P.Fitz, and J.H.Grimm, "A new view of performance analysis of transmit diversity schemes in correlated Rayleigh fading 121," *IEEE Trans.Inform.Theory*, vol. 48, no. 4, pp. 950–956, Apr. 2002.

[11] J. Wu. Ph.D. Thesis, Queens's University at Kingston, Canada, 2006.

[12] Z. Liu, "Maximum diversity in single-carrier frequency-domain equalization," *IEEE Trans.Inform.Theory*, vol. 51, no. 8, pp. 2937–2940, Aug. 2005.

[13] W. Zhang, "Comments on "Maximum diversity in single-carrier frequency-domain equalization"," *IEEE Trans.Inform.Theory*, vol. 52, no. 3, pp. 1275–1277, Mar. 2006.

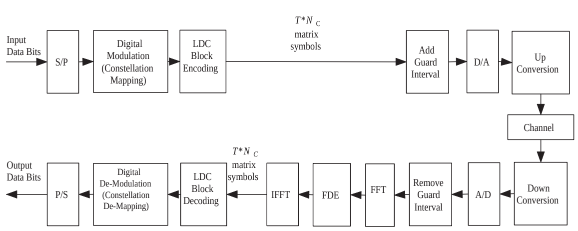

Fig. 1. Proposed LDC-CP-SCM system model

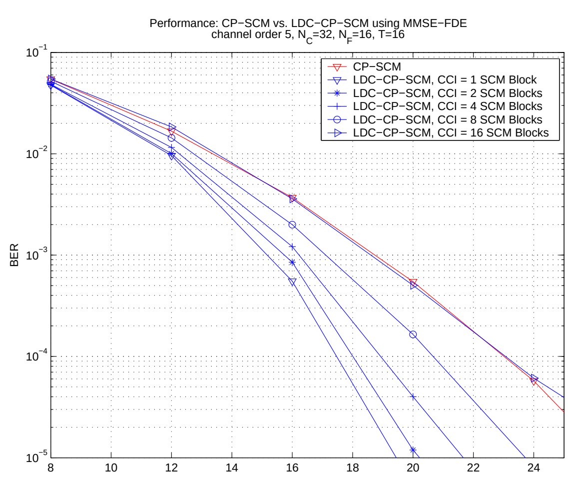

Fig. 2. BER Performance of LDC-CP-SCM vs. CP-SCM

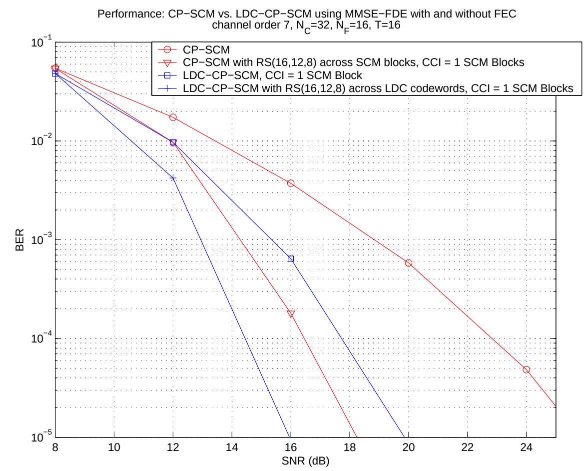

Fig. 3. BER Performance of LDC-CP-SCM with inter-LDC FEC vs. CP-SCM with inter-SCM-block FEC

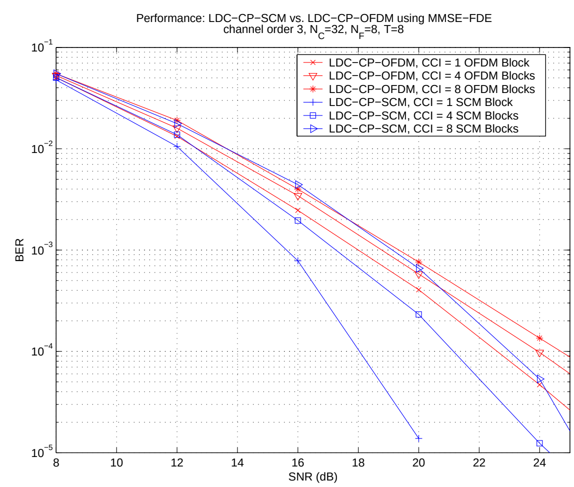

Fig. 4. BER Performance of LDC-CP-SCM vs. LDC-CP-OFDM

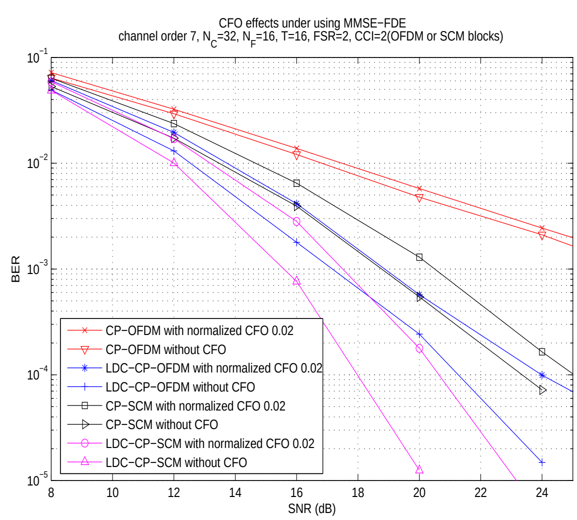

Fig. 5. BER Performance of CP based systems under CFO effects