Performance and Complexity Analysis of Eigencombining, Statistical Beamforming, and Maximal-Ratio Combining

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Abstract—For receive-side maximal-ratio combining (MRC) and maximum-average-SNR beamforming (BF), the wireless-channel fading correlation impacts symbol-detection performance — decreasing correlation improves/degrades MRC/BF performance — whereas the numerical complexity of these methods is fixed — high/low for MRC/BF. Matching signal processing complexity to the actual correlation conditions, and thus to the achievable performance, is possible with a superset of MRC and BF known as maximal-ratio eigencombining (MREC). For imperfectly known and correlated fading gains, new closed-form expressions are derived for the probability density function of the MREC output signal-to-noise ratio, as well as for the outage probability and average error probability. These new expressions permit seamless evaluation for any correlation value of MREC, MRC, and BF performance measures such as the amount of fading, deep-fade probability, diversity and array gains, and OP. Our results confirm that, in realistic scenarios, adaptive MREC can achieve MRC-like performance for BF-like complexity.

Index Terms—Adaptive antenna arrays, array gain, channel estimation, diversity gain, eigencombining, Rayleigh fading, Laplacian power azimuth spectrum, lognormal azimuth spread, numerical complexity, statistical beamforming.

I. INTRODUCTION

SMART-antenna-based wireless communications systems promise tremendous benefits in terms of data rate, user capacity, cell coverage, link quality, and transmitted signal energy [1] [2] [3] [4] [5] [6]. For receive-side smart antennas, two conventional signal processing algorithms are: 1) maximum average signal-to-noise ratio (SNR) beamforming, also referred to as statistical beamforming (BF) [7] [8, Section 9.2.2], whereby the received signal vector is linearly combined with the dominant eigenvector of the correlation matrix of the channel gain vector; and 2) maximal-ratio combining (MRC) [6] [9], whereby the received signal vector is linearly combined with the channel gain vector.

BF and MRC are designed for particular spatial fading correlation conditions and only then yield significant performance gains over the single-input single-output (SISO) transceiver. The fading gains on the different received signal branches have to be fully correlated (coherent) for BF, and uncorrelated for MRC [8, Section 9.2.2] [9] [10] [11] [12] [13] [14]. However, actual signal arrival is characterized by a Laplacian power azimuth spectrum (p.a.s.) with small-to-moderate, lognormally-distributed, and slowly-varying azimuth spread (AS) [12, Section 4.2] [14]. The resulting variable antenna correlation yields unsteady BF and MRC performance, although the numerical complexity is fixed [13, Table II, p. 922]. Whereas BF may periodically underperform, MRC may have excessive numerical complexity, which leads to inefficient hardware usage [11].

Eigencombining, also known as eigenbeamforming [15], permits matching numerical complexity to actual channel correlation, and thus to the achievable performance [11] [12] [13] [14] [16] [17]. Maximal-ratio eigencombining (MREC) actually applies the principles of both BF and MRC: first, the received signal vector is projected onto dominant eigenvectors of the channel gain correlation matrix (i.e., the Karhunen–Loeve Transform — KLT [18]); then, the resulting signals are processed according to the maximal-ratio combining criterion. The number of eigenvectors used in the KLT is referred to as the MREC order, and determines the numerical complexity of the algorithm [12] [13].

Recently, several authors have analyzed, simulated, and implemented in digital devices receive-side eigencombining [10] [11] [13] [12] [15] [19]. They compared the performance and complexity of BF, MRC, and MREC for perfectly known channel (p.k.c.), and also for imperfectly known channel (i.k.c.) whose fading gains are estimated by employing pilot-symbol-aided modulation (PSAM) at the transmitter and interpolation at the receiver [12, Sections 2.5, 3.6]. Importantly, BF and MRC were demonstrated to be performance-equivalent to special cases of MREC, which helps simplify their analysis [13] [12].

Although this paper focuses on receive-side eigencombining, i.e., on single-input multiple-output (SIMO) systems, transmit-side statistical eigenprocessing (i.e., for MISO, MIMO) has also recently been evaluated [14]. Finally, eigencombining has also been shown to benefit CDMA ‘rake’ receivers affected by high inter-tap correlation [20].

As mentioned above, MREC can be viewed as a superset of MRC and BF. For MRC in perfectly known Rayleigh fading channel, closed-form expressions for the probability density function (p.d.f.) and the cumulative distribution function (c.d.f.) of the output SNR as well as for the outage probability (OP) that appear in [3] [4] [6] [21] [22] apply for correlated channel gains, but only when the eigenvalues of the channel correlation matrix are all-distinct or all-equal.

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However, in practice, the channel fading is imperfectly known and subsets of nearly-equal eigenvalues can occur [13, Fig. 1] [12, Fig. 4.1].

For estimated fading but perfectly-known related correlation matrices, optimum (exact) MRC is described in [12, Appendix A]. Given perfect knowledge of the channel eigenstructure (i.e., eigenvectors and eigenvalues), exact MREC is described in [13, Section III.D]. A simple, yet nonclosed-form, exact-MREC average error probability (AEP) expression that covers the case when some eigenvalues may be equal appears in [13, Eqn. (7), p. 919].

For i.k.c. and correlated fading, the suboptimum (approximate) implementation of MRC described in [13, Section III.C.1] has typically been analyzed with difficulty. The performance-equivalence between this combining method and approximate MREC — which is described in [13, Section III.C.1] — has produced a valuable, yet involved, closed-form AEP expression [13, Eqn. (37)] that allows for subsets of equal eigenvalues for the channel correlation matrix.

From here on, only exact (i.e., not approximate) MREC, BF, and MRC are considered. We derive new closed-form expressions for the p.d.f. of the output SNR, as well as for the OP and AEP, that apply seamlessly (for i.k.c. and for arbitrary relative eigenvalue magnitudes, i.e., any correlation between OP and AEP, that apply seamlessly (for i.k.c. and for arbitrary for the p.d.f. of the output SNR, as well as for the eigenvalues for the channel correlation matrix.

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MEASURES FOR EXACT MREC (MRC, BF)

A. Signal, Channel, and Noise Models

Although the following analysis applies for any multibranch receiver, including the CDMA ‘rake’ [6], numerical examples are presented only for antenna arrays in frequency-flat fading channels.

Consider the signal model from [13, Section II], where the signal transmitted by a mobile station over a fading channel with Laplacian power azimuth spectrum (p.a.s.) [14, Eqn. (15)] and lognormal azimuth spread (AS) [14, Eqn. (17)] is received at the base station with an L-element antenna array. The signal model is given by the L-dimensional vector equation

\[ \tilde{y} = \sqrt{E_s} b \hat{h} + \tilde{n}, \]

where: \( E_s \) is the energy transmitted per symbol; \( b \) are equiprobable, unit-energy, M-PSK symbols; \( \hat{h} \) is the channel fading gain vector, hereafter assumed zero-mean complex Gaussian (unless stated otherwise) with correlation matrix \( \mathbf{R}_h \), i.e., \( \hat{h} \sim \mathcal{CN}(0, \mathbf{R}_h) \); \( \tilde{n} \) is the zero-mean, complex Gaussian, spatially- and temporally-white noise with variance \( N_0 \) per dimension, i.e., \( \tilde{n} \sim \mathcal{CN}(0, N_0 \mathbf{I}) \). We assume perfectly known the matrix \( \mathbf{U} \) containing the eigenvectors \( \mathbf{u}_i \), \( i = 1, \ldots, L \geq 1 : L \) of \( \mathbf{R}_h \), and the diagonal matrix \( \mathbf{A} \) formed with corresponding eigenvalues \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L \). The components of \( \tilde{y} \) are denoted hereafter as (signal) branches. The average per-branch per-symbol signal-to-noise ratio (SNR) is \( \Gamma_{s} = \frac{E_s}{N_0} \sigma^2_{h_i} \), where \( \sigma^2_{h_i} \) is the variance of the \( i \)th component of the channel gain vector, assumed to be the same on all branches for our numerical results.

B. MREC, MRC, and BF

Maximal-ratio eigencombining (MREC) of order \( N \leq L \), denoted hereafter with \( \text{MREC} \), has been described in [13, Section III.A.1] for perfectly-known channel (p.k.c.) as consisting of the following steps:

1) Karhunen–Loève Transform (KLT) the \( L \)-dimensional received signal vector \( \tilde{y} \) using the \( L \times N \) full-column-rank matrix \( \mathbf{U}_N \) formed with the first \( N \) eigenvectors of \( \mathbf{R}_h \). Mathematically, this can be written as \( y = U_N \tilde{y}, \ h = U_N \hat{h}, \ n = U_N \tilde{n} \). The components of \( y \) and \( h \) are denoted as eigenbranches and eigengains,
respectively. Since the columns of \( U_N \) are orthonormal, for \( y, h, \) and \( n \) the components are uncorrelated, and \( h \sim \mathcal{N}(0,A), n \sim \mathcal{N}(0,N_0I). \)

2) Combine the \( N \)-dimensional vector \( y \) with the eigen-gain vector \( h \), according to the maximal-ratio combining (MRC) criterion [9].

The practical case of imperfectly-known channel (i.k.c.) occurs when the fading gains are estimated, e.g., through transmitter pilot-symbol-aided modulation (PSAM), and receiver interpolation [12, Sections 2.5.1, 2.5.2, 3.6]. The numerical results shown herein employ the following estimation methods, which are detailed in [12, Section 3.6]: 1) SINC PSAM: a simple but suboptimum method wherein the interpolator vector components are computed using a sinc function; 2) MMSE PSAM: a complex but optimum method wherein the interpolator is derived according to the minimum mean-squared-error criterion.

Given eigen-gain estimates and perfect knowledge of the channel eigenstructure, implementation of optimal (exact) MREC is described in [13, Section III.D]. A comparative analysis of the performance and complexity of exact MREC, MRC, and BF is the goal in this paper. Therefore, let us now briefly review the relation of MREC with MRC and BF. MREC can be viewed as a superset of the traditional BF and MRC methods in that MREC\(_{N=1}\) represents statistical beamforming (BF) — an approach traditionally deployed at compact antennas in low-

C. Symbol-Detection Performance Measures and Analysis Methods

Let us denote with \( \gamma \) the symbol-detection signal-to-noise ratio (SNR) at the output of a signal combiner, and let \( p(\gamma) \) denote the probability density function (p.d.f.) of this SNR. The OP represents the probability that the probability of error, \( P_e(\gamma) \), exceeds a threshold [6, Section 1.1.2, p. 5], i.e.,

\[
P_e(\gamma) \triangleq \Pr[P_e(\gamma) > P_{th}]\).

If the threshold SNR, \( \gamma_{th} \), can be found so that \( P_{th} = P_e(\gamma_{th}) \) then the OP is also given by

\[
P_e(\gamma) = \Pr(\gamma < \gamma_{th}) = \int_{0}^{\gamma_{th}} p(\gamma) d\gamma,
\]

which is the cumulative distribution function (c.d.f.) of \( \gamma \) evaluated at \( \gamma_{th} \).

For M-PSK transmission and maximum-likelihood symbol detection the symbol error probability is given by [6, Eqn. 8.22, p. 198]

\[
P_e(\gamma) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \exp\left(-\frac{\gamma g_{PSK}}{\sin^2 \phi}\right) d\phi, \quad g_{PSK} = \sin^2 \frac{\pi}{M}.
\]

Then, the average error probability (AEP) [5, Eqn. 14.3-4, p. 817] [6, Eqn. 8.102, p. 219] is

\[
P_e \triangleq \int_{0}^{\infty} P_e(\gamma) p(\gamma) d\gamma
\]

\[
= \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \exp\left(-\frac{\gamma g_{PSK}}{\sin^2 \phi}\right) p(\gamma) d\gamma d\phi.
\]

\[
= \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} F_{\gamma}(\frac{g_{PSK}}{\sin^2 \phi}) d\phi,
\]

where \( F_{\gamma}(s) \triangleq E\{e^{-\gamma s}\} \) is the reversed moment generating function (r.m.g.f.) of the SNR [6, Eqn. (1.2), p. 4]. The r.m.g.f. can be readily computed for various fading types [6, Table 2.2, p. 19]. The AEP derivation approach from (7) will be adopted in Section II-F.

D. The P.D.F. of the Output SNR for Exact MREC

First, our assumptions of Gaussian noise and channel gains imply that the channel eigengains, \( h_i \sim \mathcal{N}(0,\lambda_i) \), and their PSAM-based estimates, \( g_i \sim \mathcal{N}(0,\sigma_{g_i}^2) \), are jointly-Gaussian.

Then, the exact-MREC output SNR is given by [13, Eqn. (22)]

\[
\gamma = \sum_{i=1}^{N} \gamma_i,
\]

where \( \gamma_i \) represents the SNR for the \( i \)th eigenbranch — i.e., the SNR obtained having knowledge of \( g_i \) and of the involved correlations — which is given by [13, Eqn. (19)]

\[
\gamma_i = \frac{\frac{E}{N_0} \lambda_i |\mu_i|^2}{\frac{E}{N_0} \lambda_i (1 - |\mu_i|^2) + 1} \cdot \frac{|g_i|^2}{\sigma_{g_i}^2},
\]

where \( \mu_i = \sigma_{h_i/g_i}/\sqrt{\sigma_{h_i}^2 \sigma_{g_i}^2} \) is the correlation coefficient of \( h_i \) and \( g_i \). The correlations required in (9) have been expressed for SINC and MMSE PSAM in [24, Tables 1, 2]. Eqn. (8) indicates that the combiner maximizes the MREC SNR, which justifies the title of “exact MREC”.

Since \( g_i \) is Gaussian, the eigenbranch SNR \( \gamma_i \) from (9) has exponential p.d.f.

\[
p(\gamma_i) = \begin{cases} \frac{1}{\Gamma_i} e^{-\gamma_i/\Gamma_i}, & \text{for } \gamma_i \geq 0, \\ 0, & \text{otherwise}, \end{cases}
\]

average

\[
\Gamma_i = E\{\gamma_i\} = \frac{\frac{E}{N_0} \lambda_i |\mu_i|^2}{\frac{E}{N_0} \lambda_i (1 - |\mu_i|^2) + 1},
\]

and variance [5, Eqn. (2.1-113), p. 42]

\[
\var(\gamma_i) = E\{(\gamma_i - \Gamma_i)^2\} = \Gamma_i^2,
\]

For p.k.c. we have \( \mu_i = 1 \), which reduces the SNR \( \gamma_i \) to \( \gamma_i = \frac{E}{N_0} |h_i|^2 \), and its average to \( \Gamma_i = \frac{E}{N_0} \lambda_i \).

For i.k.c., the r.m.g.f. of \( \gamma_i \) from (9) can be readily determined as [6, Table 2.2, p. 19]

\[
F_{\gamma_i}(s) \triangleq E\{e^{-\gamma_i s}\} = \int_{0}^{\infty} e^{-\gamma_i s} p(\gamma_i) d\gamma_i = \frac{1}{1 + s \Gamma_i}.
\]
Then, using the independence of \( \gamma_k, i = 1 : N, \) and (13), the r.m.g.f. of the output SNR \( \gamma \) from (8) can be written as

\[
F_{\gamma}(s) = \prod_{i=1}^{N} \frac{1}{1 + s \Gamma_i},
\]

(14)

For certain azimuth spread (AS) values some eigenvalues of \( R_h \) can become equal [13, Fig. 1]. Then, let \( \{ \Xi_1, \Xi_2, \ldots, \Xi_{N_d} \} \) denote the distinct values in the set \( \{ \Gamma_1, \Gamma_2, \ldots, \Gamma_N \} \), so that

\[
F_{\gamma}(s) = \prod_{k=1}^{N_d} \frac{1}{(s + \Xi_k)^{r_k}},
\]

(15)

where \( r_k \) denotes the algebraic multiplicity of \( \Xi_k, k = 1 : N_d \), with \( \sum_{k=1}^{N_d} r_k = N \). Applying to (15) the partial fraction expansion from [25, §2.102, pp. 56–57] yields

\[
F_{\gamma}(s) = \frac{1}{A} \prod_{k=1}^{N_d} \frac{r_k!}{(s + \Xi_k)^{r_k}},
\]

(16)

where \( A \triangleq \prod_{k=1}^{N_d} \Xi_k^{r_k} = \prod_{i=1}^{N} \Gamma_i \), and the factor \( c_{k,l} \) is given by

\[
c_{k,l} = \frac{A}{(r_k - l)!} \left\{ D_{r_k-l} \left[ F_{\gamma}(s) \left( s + \frac{1}{\Xi_k} \right) \right] \right\}_{s=-1/\Xi_k},
\]

(17)

with \( D_{r_k-l}(G(s)) \triangleq \frac{d^{r_k-l}(G(s))}{d s^{r_k-l}} \), i.e., the \( n \)th derivative of \( G(s) \). Then, \( c_{k,l} \) can be expressed in closed-form as [12, Eqn. (3.166), p. 114]

\[
c_{k,l} = (-1)^{r_k-l} \sum_{j=1}^{N_d} \prod_{j \neq k} d_j \left( \frac{1}{\Xi_j - \Xi_k} \right)^{-(r_j + i_j)},
\]

(18)

for \( k = 1 : N_d, l = 1 : r_k \), where \( \Omega_k \) stands for the set of integers \( \{ i_j, j = 1 : N_d, j \neq k \mid 0 \leq i_j \leq r_k - l, \sum_{j \neq k} i_j = r_k - l \} \), and

\[
d_j = \binom{r_j - l + i_j}{i_j}
\]

is the binomial coefficient.

Now, using the Laplace transform pair

\[
\frac{1}{(s + \Xi_k)^{r_k}} \xrightarrow{L} \frac{\Gamma_k^{r_k-1} e^{-\Gamma_k \Xi_k}}{(l-1)!},
\]

the inverse Laplace transform of (16) yields the following novel closed-form expression for the p.d.f. of \( \gamma \) for exact MREC, in the most general case when some eigenvalues can coincide:

\[
P(\gamma) = \frac{1}{A} \sum_{k=1}^{N_d} \sum_{l=1}^{r_k} c_{k,l} \cdot \frac{\Gamma_k^{r_k-1} e^{-\Gamma_k \Xi_k}}{(l-1)!}.
\]

(19)

The Appendix specializes the above derivations for channel correlation matrix with all-equal or all-distinct eigenvalues.

Since MREC_{N-1} represents BF and since MREC_{N-L} (full MREC) is performance-equivalent with MRC, Eqn. (19) also describes BF and MRC performance. Our MREC-based approach greatly simplifies MRC performance analysis for correlated and imperfectly-known channel gains. The Appendix shows how Eqn. (19) reduces to more particular expressions derived previously for MRC.

E. Exact-MREC Output SNR C.D.F. and OP

The c.d.f. of \( \gamma \) can now be obtained from (19) as

\[
P(\gamma) = \frac{1}{A} \sum_{k=1}^{N_d} \sum_{l=1}^{r_k} c_{k,l} \left[ 1 - e^{-\gamma/\Xi_k} \right]^{l} \frac{1}{(n-1)!} 
\]

(20)

Since the c.d.f. depends implicitly on \( \Gamma_s \), we will also write it as \( P(\gamma, \Gamma_s) \).

Using this c.d.f. expression, the outage probability (OP), defined in (2) can be written in closed-form for exact MREC (MRC and BF) as follows

\[
P_o = P(\gamma = \gamma_{th} | \Gamma_s).
\]

(21)

The Appendix specializes (20) and (21) to the special cases of all-equal and all-distinct eigenvalues, matching previously-derived expressions.

F. Exact-MREC AEP

Substituting (14) in the r.m.g.f.-based average error probability (AEP) derivation procedure described in Section II-C yields the following expression for the AEP of exact-MREC [13, Eqn. (28)]:

\[
P_{e,N} = \frac{1}{A} \int_{0}^{\frac{M-1}{\pi}} F_{\gamma} \left( \frac{g_{\text{PSK}}}{\sin^2 \phi} \right) d\phi = \frac{1}{A} \int_{0}^{\frac{M-1}{\pi}} \prod_{i=1}^{N_d} \left( 1 + \Gamma_s \frac{g_{\text{PSK}}}{\sin^2 \phi} \right)^{-1} d\phi.
\]

(22)

Although a nonclosed-form, this finite-limit-integral AEP expression can be readily computed. Note that (22) extends to i.k.c. the results obtained for p.k.c. in [6, Section 9.23] [14, Eqn. (82)].

A closed-form MREC AEP expression can also be derived, as follows. Using the first equality in (22) along with (16), the exact-MREC AEP expression for the most general case when some eigenvalues may be equal can be recast in the following canonical form

\[
P_{e,N} = \frac{1}{A} \sum_{k=1}^{N_d} \sum_{l=1}^{r_k} c_{k,l} \cdot \Xi_k^l \cdot I_l(\Xi_k),
\]

(23)

where

\[
I_l(\Xi_k) \triangleq \frac{1}{A} \int_{0}^{\frac{M-1}{\pi}} \left[ 1 + \Xi_k \frac{g_{\text{PSK}}}{\sin^2 \phi} \right]^{-l} d\phi
\]

(24)

is expressed in closed-form in the Appendix — see (35). Note that (23) is not as straightforward to compute as (22) is, because the factors \( c_{k,l} \) from (17) depend on the relative magnitudes of the eigenvalues of \( R_h \). The Appendix also specializes the above AEP expressions to the special cases of all-equal and all-distinct eigenvalues, matching previous results.

G. Extensions to Other Modulation, System, and Fading Cases

As indicated in [6, Chapter 8] [26, Chapter 4], the bit or symbol error probabilities, or tight bounds on them, can be written similarly to (4) for modulations other than M–PSK (e.g., M–QAM). Furthermore, the symbol-detection SNR for optimum combining can be written as a sum of possibly-correlated exponentially-distributed branch SNRs also for MISO and MIMO systems, e.g., for transmit MRC [1, Eqn.
A combiner-output-SNR expression can be recast as a sum of uncorrelated exponentially-distributed random variables with averages given by the eigenvalues of the channel gain vector correlation matrix [26, Eqn. (4.4.6), p. 53]. Then, performance measures (SNR p.d.f., OP, and AEP) for these modulations and systems can be derived as above.

Finally, for Ricean fading [6, Chapter 2] the approaches described above apply after replacing correlations with autocovariances and crosscovariances with crosscovariances. Furthermore, the p.d.f. and r.m.g.f. of the eigenbranch SNR $\gamma$ can then be derived from [6, Table 2.2, p. 19]. Therefore, an AEP expression similar to (22) is readily obtainable. However, the r.m.g.f. of the MREC output SNR can be written in a product form whose terms contain a fraction, similarly to (14), but also an exponential factor. Consequently, it is not known how this r.m.g.f. could be recast as a sum of its component terms, as in (16), and closed-form expressions for the performance measures discussed above could not be obtained.

III. PERFORMANCE EVALUATION FOR EXACT MREC VS. MRC AND BF

A. Settings for Numerical Experiments

The following performance evaluation for MREC, BF (i.e., $\text{MREC}_{N=1}$), MRC (i.e., $\text{MREC}_{N=L}$), and SISO (i.e., $\text{MREC}_{L=1}$) employs the expressions derived in Section II for the p.d.f. and c.d.f. of the output SNR, as well as for the OP and AEP. Considering a receiving uniform linear array (ULA) with $L = 5$ and normalized interelement distance $d_n = 1$ (i.e., the actual distance equals half of the carrier wavelength), and intended signal arriving with Laplacian power azimuth spectrum (p.a.s.) [14] with mean angle of arrival $\theta_0 = 0$ (the intended signal arrives from the direction perpendicular on the antenna array). The channel gains have equal variance. The maximum normalized Doppler shift (Doppler shift divided by the symbol frequency) is set to 0.01 [12, Eqn. (2.48), p. 21] [13, Table I]. Channel estimation relies on SINC or MMSE PSAM with slot length $M_s = 7$ (6 data symbols, 1 pilot symbol) and interpolator length $T = 11$ (the pilot samples from 11 surrounding slots are interpolated to yield the required estimate) [12, Section 3.6, p. 81].

B. Evaluation of the P.D.F. of the Output SNR

An $L$-element antenna array collects $L$ times more intended-signal energy than each of its constituent elements, on average over the fading. Suitable combining of the received signals can transform this additional received intended-signal energy into array gain, as discussed later. For now, of interest are only the effects of channel gain correlation and combiner type on the p.d.f. of the combiner output SNR. Thus, the numerical results discussed in this subsection have been obtained under the condition that the average output SNR for a SIMO ULA and a SISO receive antenna are equal. This is equivalent to assuming that, when an $L$-element ULA is employed, $L$ times less energy is transmitted than in the SISO case.

Fig. 1 shows several p.d.f.-related plots computed using (19), for p.k.c., vs. the output SNR, $\gamma$, and the azimuth spread (AS) [13]. The surface represents the p.d.f. of the output SNR of $\text{MREC}_{N=3}$, denoted hereafter as $p_{\text{MREC}_{N=3}}(\gamma; \text{AS})$. Also shown are the peaks of the SNR p.d.f. surfaces corresponding to SISO, BF, and MRC, and $\text{MREC}_{N=2:4}$ because they reveal the shapes of the corresponding surfaces, circumventing the need to plot each of them separately. Furthermore, the peak of the p.d.f. indicates the most likely SNR value, providing insight into the performance. Fig. 1 indicates that:

- For $\text{AS} = 0$, all combiners yield the same output-SNR p.d.f., which decreases when $\gamma$ increases. The explanation follows. For $\text{AS} = 0$ the channel gains are coherent and so $\Gamma_1 \neq 0$ and $\Gamma_2 = 0$ — see Proposition 1 in [12, p. 54] [14]. Then, $\text{MREC}_{N=2:4}$ reduces to BF, whose output SNR is exponentially distributed — see (10). The SISO and BF SNR p.d.f.s also coincide at $\text{AS} = 0$ because we assumed equal average output SNR for SISO and the SIMO ULA.
- The p.d.f. of the SISO output SNR is $\text{AS}$-independent, as expected;
- $p_{\text{SISO}, \text{BF}}(\gamma; \forall \text{AS}) \downarrow$, i.e., when $\gamma$ increases the output-SNR p.d.f. decreases, at any $\text{AS}$ value, i.e., for SISO and BF the SNR p.d.f. is a decreasing function of $\gamma$;
- $\text{max}_J [p_{\text{BF}}(\gamma; \text{AS})] \uparrow$, i.e., BF yields higher probabilities of low SNR values (worse performance) with increasing $\text{AS}$, due to increasing mismatch between the BF weights — the components of the dominant eigenvector of $R_p$, see [13, Eqn. (5), for $N = 1$] [14, Eqn. (32)] — and the corresponding channel estimation. This diminishes the array gain, as discussed later.
- For $\text{AS} \neq 0$ the SNR p.d.f. plots for $\text{MREC}_{N \geq 2}$ have different shapes than for SISO and BF, which is due to different distribution types — compare (10) and (19).
- $\text{arg max}_J [p_{\text{MREC}_{N=3}}(\gamma; \text{AS} \downarrow \text{BF})] \approx 7$ dB, because $\text{MREC}_{N=3}$ collects the intended-signal and noise energy...
along the first $N = 3$ channel eigenvectors. At very low AS, the intended-signal energy is concentrated along a single eigenvector, as shown later in the top subplot in Fig. 2 — see also [13, Fig. 1] [14, Fig. 1]. For increasing AS, the intended-signal energy is distributed more uniformly over the channel eigenvectors. Combining the corresponding (uncorrelated) channel eigengains increases the likelihood of higher SNR values. On the other hand,

\[
\arg \max_{\gamma} p_{\text{MREC},N=3}(\gamma, \text{AS} | 360^\circ ) \approx 6.25 \text{ dB}, \text{ i.e., lower than } \arg \max_{\gamma} p_{\text{MREC},N=3}(\gamma, \text{AS} | 144^\circ ) \approx 7 \text{ dB}, \text{ because for AS } \in [46^\circ, 60^\circ] \text{ eigenvalues } \lambda_1, \lambda_2, \lambda_3 \text{ convey less than } 70\% \text{ of the total average intended-signal energy, as shown in the top subplot in Fig. 2 as well as in [13, Fig. 1] [14, Fig. 1]. The remainder is loss due to mismatch between combiner weights and channel gains. This high-AS degrading effect is more pronounced for smaller } N. 
\]

- \[
\arg \max_{\gamma} p_{\text{MRC}}(\gamma, \text{AS} | 108^\circ ) \uparrow \text{, and } \\
\arg \max_{\gamma} p_{\text{MRC}}(\gamma, \text{AS} | 144^\circ ) \approx 9.25 \text{ dB}, \text{ because MRC inherently takes advantage of the intended-signal energy arriving over all } L \text{ eigenvectors.} 
\]

- Interestingly, for AS below a certain value, which increases with increasing $N$, $p_{\text{MREC},N}(\gamma, \text{AS}) \approx p_{\text{MRC}}(\gamma, \text{AS})$, i.e., MREC$_N$ can perform as well as MRC.

For i.k.c., we obtained similar p.d.f. plots as above for p.k.c. However, MMSE PSAM yields:

- \[
\arg \max_{\gamma} p_{\text{MREC},N=3}(\gamma, \text{AS} | 108^\circ ) \uparrow \text{, and } \\
arg \max_{\gamma} p_{\text{MREC},N=3}(\gamma, \text{AS} | 144^\circ ) \approx 6.25 \text{ dB,} \\
\arg \max_{\gamma} p_{\text{MREC},N=3}(\gamma, \text{AS} | 180^\circ ) \approx 5.5 \text{ dB,}
\]

implying some performance degradation compared to p.k.c. On the other hand, the simple SINC PSAM method yields a more significant deterioration:

- \[
\arg \max_{\gamma} p_{\text{MRC}}(\gamma, \text{AS} | 108^\circ ) \uparrow \text{, and } \\
\arg \max_{\gamma} p_{\text{MRC}}(\gamma, \text{AS} | 144^\circ ) \approx 3.25 \text{ dB,} \\
\arg \max_{\gamma} p_{\text{MREC},N=3}(\gamma, \text{AS} | 180^\circ ) \approx 2.5 \text{ dB.}
\]

Comparing these results with those described above for p.k.c. indicates that channel estimation results in some extra combiner–channel mismatch loss for MREC$_{N\leq L}$. Channel knowledge imperfection also deteriorates the MRC performance:

- \[
\arg \max_{\gamma} p_{\text{MRC}}(\gamma, \text{AS} | 108^\circ ) \approx 9.25 \text{ dB, } \\
\arg \max_{\gamma} p_{\text{MREC, MMSE}}(\gamma, \text{AS} | 144^\circ ) \approx 7.75 \text{ dB, and } \\
\arg \max_{\gamma} p_{\text{MRC, SINC}}(\gamma, \text{AS} | 180^\circ ) \approx 5.5 \text{ dB.}
\]

C. Amount of Fading (AF)

Given the distribution of the output SNR of a combiner, $\gamma$, one can measure fading severity by the amount of fading (AF), which is defined as [6, Eqn. (2.5), p. 18]

\[
\text{AF}(\gamma) \triangleq \frac{\text{var}(\gamma)}{\bar{E}(\gamma)^2} = \frac{E\{\gamma^2\} - (E(\gamma))^2}{(E(\gamma))^2}. 
\]

The AF describes the variability of the SNR relative to its average, and is typically independent of $E\{\gamma\}$ [6, p. 18]. The Rayleigh fading SISO case yields $\text{AF} = 1$, and thus provides a convenient reference. Smaller AF indicates less severe fading experienced at the combiner output, i.e., better performance.

For Rayleigh fading and exact MREC, the individual eigenbranch SNRs, i.e., $\gamma_i, i = 1 : L$, defined in (9) are mutually uncorrelated and exponentially-distributed random variables, with averages and variances expressed in (11) and (12), respectively. The exact-MREC output SNR defined in (8) is given by the sum of the these individual SNRs. Then, it can be shown that the AF for order-$N$ exact-MREC satisfies the following inequalities:

\[
\frac{1}{N} \leq \text{AF}(\gamma, N) = \frac{\sum_{i=1}^{N} \bar{\gamma}_i}{(\sum_{i=1}^{N} \bar{\gamma}_i)^2} \leq 1. 
\]

Based on [27, §3.2.9, p. 11], the lower bound is achieved for identically distributed eigenbranches, i.e., when $\Gamma_1 = \Gamma_2 = \ldots = \Gamma_N$. For $N = L$, this implies independent and identically distributed (i.i.d.) channel gains, based on Proposition 2 in [12, p. 54] [14]. The upper bound in (26) is achieved when $\Gamma_1 \neq 0$ and $\Gamma_2 = \ldots = \Gamma_L = 0$, which implies coherent branches, based on Proposition 1 in [12, p. 54] [14].

Fig. 2 displays vs. the AS the following: 1) in the top subplot, the correlation coefficient of the channel gains at adjacent antenna elements, $\mu_{i2}$, and the eigenvalues of $\mathbf{R}_n^\circ$, $\lambda_i, i = 1 : L$; 2) in the bottom subplot, the AF for SISO and an ULA with exact MREC (including BF and MRC), for SINC PSAM. Note first, that due to the continuous changes in the relative magnitudes of the eigenvalues, this figure could not have been obtained using previously-available SNR p.d.f. expressions [3] [4]. Then, note that, as SISO, BF yields $\text{AF} = 1$, i.e., no fading reduction. On the other hand, the fading-reduction capabilities of MREC$_{N=2}$ equal those of MRC for $\text{AS} \leq 6^\circ$, and improve further before reaching a floor for $\text{AS} \approx 25^\circ$. This floor is inversely proportional to the MREC order, which is in agreement with the lower bound in (26). The intuitive explanation is that, for large AS, MREC combines $N$ uncorrelated channel eigengains of similar strengths, thus reducing fading $N$-fold — see also Section 1 of the Appendix.

Fig. 3 shows 1/AF computed using (26) for exact MREC$_{N=3}$ with SINC PSAM. Similar (not shown) results have been obtained for MMSE PSAM. Very small AS yields $\text{AF} \approx 1$, reflecting a lack of diversity, whereas AS $\gamma$ yields $\text{AF}^{-1}$, at any value of $\Gamma_s$. 


Fig. 4 plots the output-SNR c.d.f. computed using (20) for exact MREC$_{N=3}$, SINC PSAM, unit-variance channel gains (assumed hereafter for ULA and SISO), and $\Gamma_s = E_s/N_0 = 10$ dB. The slope of $P(\gamma, \text{AS} = 0)$ is unitary (also for all other MREC orders, as other results, not shown here, have indicated). On the other hand, the slope of $P(\gamma, \text{AS} = 20^\circ)$ increases. For large AS this slope becomes proportional to the MREC order, $N$.

The c.d.f. of the combiner output SNR can be used to evaluate receiver performance in fading channels through the deep-fade probability [23, p. 55]. A deep fade is defined as the situation in which, although $\Gamma_s$ is large, the output SNR is subunitary (i.e., below 0 dB). Table I shows this probability for exact BF, MREC$_{N=3}$, and MRC, for $\Gamma_s = 30$ dB and AS = 0, 10°, 20°. For AS = 0, BF, MREC, and MRC coincide. With
increasing AS, BF performance degrades, whereas MREC and MRC yield significant improvements. MREC can actually yield a significant proportion of the MRC performance.

Fig. 5 displays the exact-MREC_{N=3} OP computed using (21) for MMSE PSAM and QPSK threshold error probability $P_{e, th} = 10^{-2}$, which, based on (4), corresponds to $\gamma_s \approx 8.2$ dB. At high $\Gamma_s$, the magnitude of the slope of $P_0(\Gamma_s)$ is 1 for $\text{AS} = 0$, and increases with increasing $\text{AS}$, reaching a maximum of 3 for $\text{AS} \approx 20^\circ$. This and other (not shown) results reveal that, at high AS, the high-$\Gamma_s$ OP slope magnitude is given by the MREC order. Furthermore, the minimum AS that maximizes the slope magnitude increases with increasing MREC order. The high-$\Gamma_s$ slope of symbol-detection performance measures reveals the benefits of diversity combining in fading channels, and is often referred to as the diversity order [26].

E. Array Gain, Mismatch Loss, and Diversity Gain

The amount of fading, deep-fade probability, and the diversity order are qualitative indicators of the fading-reduction capabilities of combiners. It would be more practical be to determine, as shown next, how much energy the mobile station can save when the base station receiver employs a multibranch combiner compared to the SISO case. Towards this end, the AEP expression from (22) is inverted numerically to determine the $\Gamma_s$ values necessary for MREC and SISO to achieve a given AEP. Their difference

$$\Delta \Gamma_s = \Gamma_{s, \text{SISO}} - \Gamma_{s, \text{MREC}} \quad \text{[dB]} \quad (27)$$

is the SNR gain of MREC over SISO.

Fig. 6 shows this gain vs. AEP and AS for exact MREC_{N=3}, for QPSK and MMSE PSAM. Note first that for $\text{AS} = 0$ the AEP level does not affect $\Delta \Gamma_s$. This is because only one channel eigenvalue is then non-zero [12, Proposition 1, p. 54] [14, Proposition 1], and thus the corresponding eigenvector conveys all the intended-signal energy. Therefore, MREC_{N=3} reduces to BF in this case. The output-SNR distribution is then exponential for both MREC and SISO. Thus, for $\text{AS} = 0$,...
MREC can only provide an average-output-SNR gain over SISO. Such increase is commonly known as array gain [1, Sections 1.2.2, 5.2].

Let us investigate further the MREC array gain. Using (8), the average output SNR for exact MREC $N$ can be written as

$$\Gamma = E\{\gamma\} = \sum_{i=1}^{N} \Gamma_i,$$

(28)

where $\Gamma_i > 0$, $i = 1 : N$, are the eigenbranch average SNRs, defined in (11). Then, the array gain is the ratio of the MREC and SISO average output SNRs, i.e.,

$$AG_{\text{MREC}} = \frac{\Gamma}{\Gamma_0},$$

(29)

which, for identically-distributed and perfectly-known channel gains, can be written as

$$AG_{\text{MREC}} = 10\log_{10} \left( \frac{\sum_{i=1}^{N} \lambda_i}{\sum_{i=1}^{L} \lambda_i} \right) \epsilon_{[10\log_{10} N, 10\log_{10} L]}.$$

The upper bound is achieved for coherent channel gains, i.e., for $\text{AS} = 0$ the MREC array gain is proportional to the number of branches, $L$. Fig. 6 actually reveals that $\Delta\Gamma_s \approx 7 \pm 3$ dB at $\text{AS} = 0$.

The MREC array gain lower bound from (30), i.e., $10\log_{10} N$, is achieved for uncorrelated channel gains. Low correlation is incurred for large AS, as shown in the top subplot in Fig. 6. Thus, when AS increases from zero to large values, $AG_{\text{MREC}}$ diminishes from $10\log_{10} L$ to $10\log_{10} N$.

Let us first consider the case of BF, i.e., $MREC_{N=1}$ Then, the array gain decreases from $10\log_{10} L$ to 0. This is verified in Fig. 7, which displays contours obtained by slicing at $\text{AEP} = 10^{-3}$ surfaces as that shown in Fig. 6 for all $N = 1 : L$. Recall that BF does not offer fading reduction over SISO. Thus, the $\Delta\Gamma_s$ plot shown in Fig. 7 for BF actually represents array gain only. For $\text{AS} = 0$ the BF weight vector (the dominant eigenvector) is perfectly aligned with the channel gain vector, and thus BF maximizes the array gain. With increasing AS there is an increasing mismatch between the BF weights and the channel gains. The mismatch loss will diminish and ultimately cancel the BF array gain.

Also for $MREC_{1<N<L}$ there can be noticeable mismatch loss, as Fig. 7 indicates. Actually, after $\Delta\Gamma_s$ peaks at an AS value proportional to $N$, $\Delta\Gamma_s$ suffers a reduction inversely-proportional to $N$. The explanation for the mismatch loss in $MREC_{1<N<L}$ is a simple generalization of the explanation given above for BF. Assuming p.k.c., the $MREC_{N=L}$ weight vector with respect to the received signal vector $\tilde{y}$ can be shown to be $U_N \tilde{y}$, i.e., nonidentical to the channel vector, $\tilde{h}$, which would be required for perfect combining. Nonetheless, given $N$, these two vectors nearly coincide for AS below
the value for which most of the intended-signal energy arrives along the \( N \) selected eigenvectors. For higher AS, intended-signal energy arrives from directions unaccounted for by the weight vector deployed by MREC\(_N\), yielding higher mismatch loss. Higher \( N \) ensures that this mismatch loss begins affecting performance at higher AS, and that the loss is smaller. Clearly, there is no such mismatch loss for full MREC (MRC) because the receiver weight vector matches the channel vector. This is confirmed by the line corresponding to MRC in Fig. 7.

Figs. 6 and 7 show that \( \Delta \Gamma \) initially increases with AS increasing from zero, which represents diversity gain [1, Sections 1.2.2, 5.2]. Increasing AS yields decreasing channel gain correlation, and thus the peaks of the SNR p.d.f. occur at larger SNR values — see Fig. 1. This reduces the amount of fading, the deep-fade probability, and increases the magnitude of the slope of AEP(\( \Gamma_x \)), i.e., the diversity order. In Fig. 6, the SNR is larger for smaller AEP because the AEP slopes for MREC\(_N=3\) and SISO are 3 and 1, respectively, which means that the AEP decreases much faster with \( \Gamma_x \) for MREC\(_N=3\) than for SISO.

Fig. 7 indicates that for AS < 4\( ^\circ \), MREC\(_N=2\) generates almost the same SNR gain as MREC\(_N=3\) (which includes MRC). Furthermore, for AS < 10\( ^\circ \), MREC\(_N=3\) generates almost the same SNR gain as MREC\(_N=4.5\). Finally, for AS < 18\( ^\circ \), MREC\(_N=4\) generates almost the same SNR gain as MRC. On the other hand, throughout the shown AS range, MREC\(_N>1\) greatly outperforms BF.

IV. PERFORMANCE AND COMPLEXITY COMPARISON OF BF, MRC, AND ADAPTIVE MREC, FOR RANDOM AZIMUTH SPREAD

The above numerical results suggest that MREC performance can be maximized by suitable selection of its order, \( N \). However, the order impacts MREC numerical complexity, which translates into certain baseband processing resource requirements [11] [12, Chapter 5]. The numerical complexity of MRC and MREC\(_N\), in terms of the number of complex multiplications and additions required per detected symbol for interpolation, combining, and (only in MREC) for KLT, are reproduced in [12, Table 3.7, p. 132] [13, Table II]. We recently found that MREC performance does not degrade significantly when the channel eigenstructure is updated by feeding received-signal samples into the deflation-based projection approximation subspace tracking (PASTd) algorithm [17]. Furthermore, PASTd-based eigenstructure tracking does not increase the complexity of adaptive MREC significantly because of PASTd’s simplicity and because the eigenstructure changes very slowly (see [13, Eqn. (3)]).

Previous studies have indicated that exact and approximate implementations of MREC with SINC and MMSE PSAM have similar complexities, due to thedecorrelating effect of the KLT — see [12, Section 3.12] [13] [16] [17]. On the other hand, MRC can have much higher computational complexity for exact vs. approximate combining, and for MMSE vs. SINC PSAM estimation.

Fig. 8 plots the numerical complexities computed from [13, Table II] for SISO, BF, MREC\(_N=1:5\), and MRC, for \( L = 5 \), interpolator length \( T = 11 \), SINC and MMSE PSAM, and exact combining. Note that for MMSE PSAM, MRC is much more complex than even full-MREC, because the channel gain vector required for MRC is estimated using matrix operations [12, Eqn. (3.113), p. 83], whereas the eigengains required for MREC are estimated separately, as vectorial inner products [12, Eqn. (3.107), p. 82]. On the other hand, SINC PSAM inherently separates the estimation of the components of both the channel gain and eigengain vectors, so that the complexities of exact MRC and full-MREC are similar.

To minimize MREC complexity and achieve near-MRC performance we apply the simple and effective bias–variance tradeoff criterion (BVTC) [11] [12, Section 5.2.2] [13] [17] [18] for exact-MREC order selection. For realistic performance and complexity comparisons, a sufficient number (10000) of independent lognormal AS samples for a typical urban scenario [13, Section II] have been generated with [13, Eqn. (2)]. The produced random AS sequence had an average of 9.81\( ^\circ \), a standard deviation of 13.4\( ^\circ \), and \( \Pr(1^\circ < AS < 20^\circ) \approx 0.83 \). For exact SISO, BF, MRC, and adaptive MREC with MMSE PSAM, Fig. 9 shows the means over the AS samples of the outage probabilities for QPSK — computed with (21), for \( P_{e, th} = 10^{-2} \), or, equivalently, for \( \gamma_{th} = 8.2 \) — and of the computational complexities (computed from [13, Table II]). For reference, the lower subplot also displays a scaled version of the MREC order output by the BVTC, i.e., 10\( ^{\circ} \). Note that, since the numerical complexities of SISO, BF, and MRC are \( N \)- and \( \Gamma_x \)-independent, they have not required averaging. On the other hand, for MREC, the order output by the BVTC (and thus the numerical complexity) depends on AS and \( \Gamma_x \). Fig. 9 indicates that the BVTC MREC order increases with increasing \( \Gamma_x \), which improves MREC performance but also increases complexity.

Fig. 9 shows that throughout the SNR range, adaptive
MREC yields MRC-like performance, which can be much better than for BF due to diversity gain and additional array gain. The outage probability performance level $P_o = 10^{-1}$, for instance, is achieved with SISO for $\Gamma_s \approx 22$ dB and with BF for $\Gamma_s \approx 16$ dB. Thus, BF produces an array gain of about 6 dB over SISO, which is about 1 dB lower than that achievable in ideal conditions, i.e., $10 \log_{10} L_{\Gamma = 5} \approx 7$ dB, for $L = 5$ coherent and perfectly known channel gains. MRC and BVT-based adaptive MREC yield an extra 4 dB gain. At $\Gamma_s = 12$ dB, the BVTC outputs an average order of $N_{avg} \approx 2.86$ for MREC. Then, based on [13, Table II], we found that for MMSE PSAM, BVT C MREC is 2.86 and 3.96 times more complex than BF and SISO, respectively, but 5.92 times less complex than MRC.

For SINC PSAM and the same random AS sequence, other (not shown) numerical results have indicated that $P_o = 10^{-1}$ is achieved with MRC and with BVTC-based adaptive MREC of average order $N_{avg} \approx 3.02$ for $\Gamma_s \approx 14$ dB, with BF for $\Gamma_s \approx 18$ dB, and with SISO for $\Gamma_s \approx 24$ dB. Thus, the relative performances of these combining methods has not changed compared to the case of MMSE PSAM. However, while $P_o = 10^{-1}$ is achieved for MMSE PSAM with adaptive MREC with $N_{avg} \approx 2.86$ at $\Gamma_s \approx 12$ dB, SINC PSAM requires $N_{avg} \approx 3.02$ at $\Gamma_s \approx 14$ dB, i.e., not only a 2 dB performance loss but also an increase of about 6% in complexity. SISO and BF experience the same performance loss but no complexity change for SINC PSAM vs. MMSE PSAM. On the other hand, exact MRC experiences a 2 dB performance loss compensated by a 3.6-fold complexity reduction with SINC PSAM compared to MMSE PSAM.

Numerical results analogous to those from Fig. 9 have appeared for the AEP, based on analysis and simulations, in [12, Fig. 4.5, p. 149] [13, Fig. 2]. Results that show the AEP vs. time for temporally-correlated samples of lognormal AS appear in [11, Figs. 5] [12, Fig. 4.6, p. 151] [13, Fig. 3] [17, Figs. 4, 5]. In [12, Chapter 5] [11] we demonstrated for fixed-point FPGA-based implementations that the lower average complexity of adaptive MREC vs. MRC translates into reduced processing resource requirements and power consumption for the former. It has also been found that adaptive MREC could benefit from the AS independence among subscriber stations [11, Figs. 5–9] [12, Figs. 5.5–5.15, pp. 170–187]. Subscribers experiencing high AS can be allocated more base-station eigenprocessing modules [11, Section 2.5, p. 4] [12, p. 170] for better performance, which yields azimuth spread diversity.

V. SUMMARY AND CONCLUSIONS

This contribution completes earlier performance and numerical complexity studies of receive-side optimum (exact) maximal-ratio eigencombining (MREC) vs. statistical beamforming (BF) and maximal-ratio combining (MRC). A novel closed-form expression has been derived for the p.d.f. of the output signal-to-noise ratio of MREC that applies seamlessly for perfectly or imperfectly known channel gains and arbitrary fading correlation value, i.e., any combination of array interelement distance, mean angle of arrival, power azimuth spectrum (p.a.s.) type, and azimuth spread (AS) value. This expression also applies to BF and MRC for smart antenna receivers as well as to MRC and MREC for correlated ‘rake’ receiver taps in CDMA systems. This p.d.f. expression has been used to derive widely-applicable closed-form expressions for the outage probability (OP) and the average error probability (AEP) of MREC, BF, and MRC. The advantages of MREC over BF and MRC have been analyzed in terms of the amount of fading, deep-fade probability, diversity and array gain, OP, and numerical complexity. For realistic (Laplacian) base-station p.a.s. and lognormal AS, optimum channel fading estimation, and optimum combining implementation, adaptive MREC has been shown to achieve near-optimum average performance at a fraction of the MRC complexity. Thus, adaptive MREC emerges as a practical alternative to the conventional, BF and MRC, approaches, for smarter antennas as well as for more efficient CDMA receivers.

APPENDIX

SPECIAL-CASE CLOSED-FORM EXPRESSIONS FOR PERFORMANCE MEASURES FOR EXACT MREC

1) The Case when All Eigenvalues are Equal: In this case, $\Gamma_s \triangleq \Gamma, i = 1 : N$, and the r.m.g.f. expression from (14) reduces to [3, Eqn. (5.2-14), p. 319] [4, Eqn. (10-61), p. 310]

$$F_{\gamma}(s) = \frac{1}{(1+ s \Gamma)^N},$$

(31)

whose inverse Laplace transform is

$$P(\gamma) = \frac{\Gamma^{N-1} e^{-\gamma/\Gamma}}{(N-1)! \Gamma^N}.$$  

(32)

Substituting this into the outage probability (OP) definition from (3) yields for exact MREC the following closed-form OP expression [3, Eqn. (5.2-15), p. 319] [4, Eqn. (10-64), p. 310]:

$$P_o = 1 - e^{\gamma_{th}/\Gamma} \sum_{n=0}^{N-1} \frac{1}{n!} \left(\frac{\gamma_{th}}{\Gamma}\right)^n.$$  

(33)
Furthermore, with the notation $\Delta \Gamma_{\text{GPSK}}$, the finite-limit integral exact-MREC AEP expression for M–PSK from (22) becomes

$$P_e = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{1}{\sin^2 \phi} \right)^{-N} d\phi.$$  (34)

With the notations $\xi = \sqrt{\frac{1}{M-1}}$, $\alpha = \xi / \tan \frac{\pi}{M}$, and $\varphi = \tan^{-1} \alpha$, the above can be written in closed-form as [6, Eqns. 5A.17–19, pp. 127–8]

$$P_e = \frac{M-1}{M} - \frac{\xi}{\pi} \sum_{n=0}^{N-1} \left(\frac{2n}{n} \left(1 - \frac{\xi^2}{2} \right)^n \right) \times \left\{ \frac{\pi}{2} + \varphi + \sin \varphi \sum_{i=1}^{n} \frac{4^{n-i}}{(2n-i)} \right\}$$  (35)

where $^{a\choose b}$ represents the binomial coefficient, i.e., $^{a\choose b} = \frac{a!}{(a-b)!b!}$, and $a! \triangleq a \cdot (a-1) \cdot (a-2) \cdots 3 \cdot 2 \cdot 1$. BPSK implies $\varphi = 0$, and so (35) reduces to Proakis’ expression [5, Appendix C, Eqn. C-18, p. 955]

$$P_e = \frac{1}{2} \left[1 - \xi \sum_{n=0}^{N-1} \left(\frac{2n}{n} \left(1 - \frac{\xi^2}{2} \right)^n \right) \right],$$  (36)

which can also be written as [6, Eqn. 5A.4b, p. 125]

$$P_e = \left[\frac{1}{2} \left(1 - \xi \right)\right]^{N-1} \sum_{n=0}^{N-1} \left(1 + \frac{1}{n} \right)^{-n} \frac{1}{2} \left(1 + \xi \right)^{n}.$$  (37)

2) The Case when All Eigenvalues are Distinct: In this case, $N_0 = N$ and $r_k = 1, \forall k = 1 : N$, so that (16) and (17) yield

$$F_\gamma(s) = \sum_{i=1}^{N} c_i 1 + s \Gamma_i.$$  (38)

where

$$c_i = \prod_{j \neq i}^{N} \frac{\Gamma_i}{1 - \Gamma_i}.$$  (39)

Then, the p.d.f. of $\gamma$, i.e., the inverse Laplace transform of $F_\gamma(s)$, has the simple closed-form expression

$$p(\gamma) = \sum_{i=1}^{N} c_i \frac{1}{\Gamma_i} e^{\gamma / \Gamma_i}.$$  (40)

For p.k.c. the above reduces to [4, Eqn. 10-60, p. 308]. The OP for M–PSK and exact MREC is, from (3) and (40),

$$P_\emptyset = \sum_{i=1}^{N} c_i \left(1 - e^{-\gamma_0 / \Gamma_i} \right).$$  (41)

The AEP is obtainable as follows, using (23) and (24):

$$P_e = \frac{1}{2} \sum_{i=1}^{N} c_i \left(1 - \sqrt{\frac{\Gamma_i \Delta \Gamma_{\text{GPSK}}}{\Gamma_i \Delta \Gamma_{\text{GPSK}} + 1}} \right).$$  (42)

For BPSK this reduces to [19, Eqn. (16)]. For BPSK, p.k.c., and uncorrelated branches, (42) reduces to the ideal-MRC AEP-formula proposed by Proakis in [5, Eqn. 14.5-28, p. 847].

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