Optimization of rateless-coded asynchronous multimedia multicast

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***Abstract*—We investigate the optimization of asynchronous multimedia multicast to heterogenous users. A priority encoding transmission (PET) based packetization scheme in combination with rateless coding has recently been proposed for multicasting. In previous work, a suboptimal greedy search algorithm is proposed to find the allocation of source symbols over different layers. However, the complexity of the algorithm is high for large numbers of user classes and the optimality of the algorithm is not proven. In this paper, we first show that the problem can be transformed to an equivalent convex optimization problem under the relaxation of integer constraints. In addition, a solution is found analytically for the optimal allocation. Numerical results demonstrate that the optimal analytical solution matches the results using the greedy search algorithm as well as results obtained using convex optimization software.**

I. Introduction

Multimedia multicast is gaining increasing attention due to the increasing demand for multimedia content on mobile devices. Many multimedia multicast applications, e.g. media-on-demand services, aim to provide asynchronous access to multiple users, i.e., users can tune-in the multicast session at arbitrary times while are able to view the media from the start.

Traditionally, Reed-Solomon (RS) codes are used for correcting packet erasures in multimedia applications. However, RS codes, like most fixed-rate forward error correction (FEC) codes, lack the design flexibility to cope with the different loss rates and access times of heterogenous users. Recently proposed fountain codes [1], such as LT [2] and raptor codes [3], have been shown to be strong candidates of application-layer FEC codes for asynchronous multicast applications [4]. This is mainly due to the rateless property of fountain codes. In order to decode fountain codes, it does not matter which fountain encoded symbols are received as long as enough encoded symbols have been collected by the receiver.

In addition to different packet loss rates, heterogeneous users may also have different quality of service (QoS) requirements for the play-back of multimedia content. On the source coding side, scalable video coding (SVC) has been designed to allow for progressive reconstruction of multimedia content at the receiver. Hence, source symbols are created with different importance. Therefore, an important challenge in deploying fountain codes as FEC codes for multimedia multicast is to provide unequal error protection (UEP) for the source symbols.

In [5] and the references therein, rate-distortion optimization for multimedia multicast is explored using fixed-rate RS codes. However, these papers focus on best-effort QoS and do not allow for asynchronous user receptions. In [6], [7] and [8], fountain codes with UEP properties are proposed. However, none of these approaches provide asynchronous reception to heterogeneous users with differing QoS guaranties.

The asynchronous multimedia multicast optimization problem with heterogenous QoS constraints is first investigated in [9]. In [9], a priority encoding transmission (PET) based packetization scheme [10] using rateless codes is proposed. The results in [9] show that for multimedia multicast, the proposed packetization scheme using rateless codes are superior to traditional schemes with fixed-rate codes in terms of both performance and design flexibility. However, in solving the optimization problem of asynchronous multimedia multicast, [9] uses a suboptimal greedy search method which has high complexity. In addition, the optimality of the greedy search algorithm is not proven and the algorithm does not provide the same insights as analytical solutions.

In this paper, we further investigate the asynchronous multicast optimization problem and find a systematic and low complexity solution. We show that under the relaxation of integer solution constraints, this problem can be transformed into a convex optimization problem. Furthermore, an analytical solution is found. Numerical results using both the analytical method and that obtained using convex optimization software [11] [12] are shown [1].

The rest of the paper is organized as follows: Section II describes the system setup, transmission scheme proposed in [9], and the original problem formulation; Section III transforms the formulation in Section II to an equivalent but simplified convex optimization problem by reducing the number of parameters; Section IV presents analytical solutions to the convex optimization problem formulated in Section III; Section V provides numerical results to verify our findings; Section VI provides the conclusions of this paper.

II. Asynchronous multicast system setup and problem formulations

A. System setup and the PET packetization scheme

In this paper, we consider a problem where the server is multicasting a scalable multimedia source, e.g., image or video to heterogenous users. The users are divided into a total of J classes according to their QoS requirements. The QoS requirements of Class j users are characterized by a target reconstruction quality of the source, measured by the peak

[1] In [13], we also investigate a more general asynchronous mutlicast problem where there are transmission deadlines and outage probability constraints for each user class, which is not presented here due to space limitations. We show in [13] that the problem with outage constraints is also a convex optimization problem and we find an analytical solution for that problem when there are two user classes.

signal-to-noise ratio (PSNR) threshold γ_j. Without loss of generality, it is assumed that $\gamma_1 \leq \gamma_2 ... \leq \gamma_J$. Each class of users also has different channel qualities. It is assumed that Class j users experience a packet erasure channel with erasure rate σ_j, where at any given time σ_j is a random variable with probability density function (PDF) denoted by $f_{\sigma_j}(.)$. However, the value of σ_j does not change during one code frame. This usually corresponds to a block fading channel model used in the physical layer.

The scalable source symbol stream is encoded and loaded into packets using a priority encoding transmission (PET) based scheme before multicasting to heterogenous users. The PET packetization scheme is first proposed with RS codes in [10]. Later, [9] proposes a transmission scheme to combine the PET packetization scheme with rateless codes. The detailed structure of the PET packetization with rateless coding is shown in Fig. 1. The source symbol stream is first partitioned into L vertical layers such that source symbols with a lower layer index are more important than those with a higher layer index, where L is the total number of layers, which is equal to the packet length. The number of source symbols inside Layer l, $l \in \{1, 2, ..., L\}$ is denoted by K_l. The K_l source symbols are then encoded by a rateless encoder to produce arbitrary number of rateless encoded symbols $d_{l1}, d_{l2}, ..., d_{li}, ...$, where d_{li} represents the $i-th$ rateless encoded symbol in the $l-th$ layer. Each data packet is formed by selecting one corresponding encoded symbol from each layer, i.e., Packet i consists of symbols $d_{1i}, d_{2i}, d_{3i}, ..., d_{Li}$. The PET packetization design objective is to ensure that the information symbols in a layer with smaller index are better protected against packet loss compared to information symbols in a layer with a larger index. Therefore, the following constraints have to be satisfied:

$$K_1 \leq K_2 \leq \cdots \leq K_L. \tag{1}$$

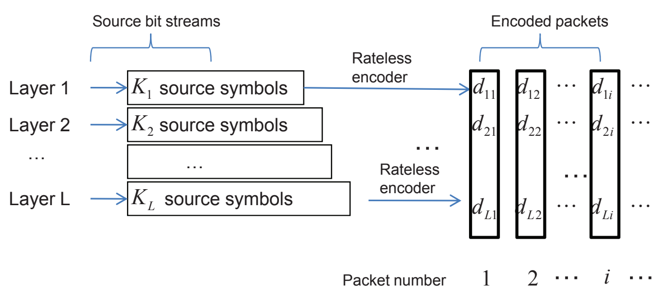

Fig. 1. The PET packetization scheme combined with rateless coding.

B. Users' QoS requirements and cost function

For a given source coder, the PSNR or equivalent fidelity measures are a function of the number of decoded symbols, where the symbols are in decreasing order of importance. The QoS requirement of a Class j user is that the obtained PSNR value should be greater than or equal to QoS threshold γ_j, which can be expressed as:

$$PSNR_j(K_1, K_2, ..., K_L) \geq \gamma_j, \tag{2}$$

where $PSNR_j(K_1, K_2, ..., K_L)$ represents the PSNR of Class j users given the allocated set of $K_1, K_2, ..., K_L$. Let $f(.)$ represent the PSNR value as a non-decreasing function of the total number of symbols decoded by the receiver, h_j denote the number of layers (starting from the first layer) that a Class j user needs to decode to achieve the target PSNR threshold γ_j for a given set $\{K_1, K_2, ..., K_L\}$. Then the QoS requirements can be expressed as:

$$\sum_{l=1}^{h_j} K_l \geq f^{-1}(\gamma_j). \tag{3}$$

where f^{-1} denotes the inverse function of f.

In the asynchronous multicast problem, the objective function is to minimize the average time that all users spend collecting rateless coded packets until their QoS requirements have been met, which is proportional to the average number of transmitted packets required to meet each user's QoS constraint, or equivalently, to successfully decode Layer h_j. The average number of transmitted packets required to meet the QoS constraint of a Class j user given the erasure rate σ_j is given by [9]:

$$E(M_j) = \frac{K_{h_j}(1+\omega)}{1-\sigma_j}, \tag{4}$$

where M_j is the number of transmitted packets required to meet Class j user's QoS requirement, $E(.)$ denotes expectation, h_j is the number of decoded layers required to achieve Class j users' QoS requirements and ω is the overhead of the rateless code, which is assumed to be fixed for different realizations. For analytical simplicity, the ceiling operator on $K_{h_j}(1+\omega)$ is omitted in the cost function.

In the case σ_j is a random variable with a PDF $f_{\sigma_j}(.)$,

$$\begin{aligned} E(M_j) =& E_{\sigma_j}\left(E(M_j)|_{\sigma_j}\right) \\ =& K_{h_j}(1+\omega)E\left(\frac{1}{1-\sigma_j}\right) \\ =& K_{h_j}(1+\omega)\int_0^1 \frac{f_{\sigma_j}(x)}{1-x}dx. \end{aligned} \tag{5}$$

Finally, the cost function for all the user classes is given by

$$\begin{aligned} M_{av} =& \sum_{j=1}^{J} w_j E(M_j) \\ =& (1+\omega)\sum_{j=1}^{J} w_j K_{h_j} \int_0^1 \frac{f_{\sigma_j}(x)}{1-x}dx \\ =& \sum_{j=1}^{J} \eta_j K_{h_j}, \end{aligned} \tag{6}$$

where $\eta_j \equiv w_j(1+\omega)\delta_j, j = 1, 2, ..., J$ are the combined cost coefficients of Class j users, which combine the importance weighting coefficient w_j, $\sum_{j=1}^{J} w_j = 1$, code-efficiency factor $(1+\omega)$ and channel quality factor

$$\delta_j \equiv \int_0^1 \frac{f_{\sigma_j}(x)}{1-x}dx. \tag{7}$$

C. Optimization problem formulation

In summary, the asynchronous multicast optimization problem is to find the best non-decreasing set $\{K_1, K_2, ..., K_J\}$ to minimize Eq. (6), such that users' target PSNR is met. This problem can be summarized as:

Problem 1:

$$\min_{K_1,...,K_L} \quad M_{av} \tag{8}$$

subject to

$$K_1 \leq K_2 \leq \cdots \leq K_L \tag{9}$$

D. Greedy search algorithm

A direct way to solve Problem 1 is to list all possible sets of source symbol layer allocations $K_1, K_2, ..., K_L$, test the constraints and compare the resultant cost. The optimal solution is the set $\{K_1, K_2, ..., K_L\}$ which provides the minimal cost among those that satisfy the constraints. However, the complexity of such a brute-force search algorithm can be prohibitively high as it depends on the total number of source symbols $f^{-1}(\gamma_J)$ as well as L. A suboptimal greedy search algorithm is proposed in [9]. The basic idea of this algorithm is to first find all possible sets of $\{h_1, h_2, ..., h_J\}$, which represents the number of layers required to decode for each user class. For each predetermined allocation of $\{h_1, h_2, ..., h_J\}$, the greedy search algorithm allocates $\lceil f^{-1}(\gamma_j)\rceil - \lceil f^{-1}(\gamma_{j-1})\rceil$ source symbols to layers $h_{j-1}+1$ to h_j *nearly equally*, where $\lceil f^{-1}(\gamma_0)\rceil = 0$. By *nearly equally* we mean the difference among the number of source symbols in Layers $h_{j-1}+1$ to h_j, i.e., $K_{h_{j-1}+1}, K_{h_{j-1}+2}, ..., K_{h_j}$, should be at most 1. In the end, the best possible allocation is chosen by selecting the one with minimal cost that satisfies the outage constraints.

The greedy search algorithm proposed in [9] has a complexity of roughly $JL^{J-1}/2$ elementary operations. Although the complexity is lower compared to brute force search over all possible sets of $\{K_1, K_2, ..., K_L\}$, it is still very high for large L and J. In addition, the optimality of the greedy search algorithm has not been proven. This motivates a lower complexity and analytical solution to this problem.

III. Transformation to an Equivalent and Simplified Convex Optimization Problem

A. Grouping layers to chunks

To find an analytical solution to Problems 1, we first divide the source symbols in each block into J chunks, where the number of symbols in each chunk is given by

$$U_j = \lceil f^{-1}(\gamma_j)\rceil - \lceil f^{-1}(\gamma_{j-1})\rceil \quad j = 1, 2, ..., J, \tag{10}$$

where γ_j is the target PSNR threshold for Class j users, $f^{-1}(x)$ is the minimum number of decoded symbols required to reach the target PSNR threshold x and $\lceil f^{-1}(\gamma_0)\rceil = 0$. Define

$$l_j = h_j - h_{j-1} \qquad j = 1, 2, ..., J, \tag{11}$$

where $h_j, j = 1, 2, ..., J$ is the number of required decoded layers to reach target QoS of Class j users and $h_0 = 0$. Therefore, l_j represents the number of layers allocated to Chunk j, which should satisfy

$$\sum_{j=1}^{J} l_j = h_J = L. \tag{12}$$

In addition, we have

$$\sum_{i=h_{j-1}+1}^{h_j} K_i = U_j \qquad j = 1, 2, ..., J. \tag{13}$$

B. Reducing the number of parameters

Problem 1 contains L variables. In order to obtain an equivalent problem with fewer variables, we first prove the following fact.

Lemma 1: If we relax the integer constraints on K_{h_j}, then the optimal solution to Problem 1 should satisfy $K_{h_{j-1}+1} = K_{h_{j-1}+2} = ... = K_{h_j}$ for $j = 1, 2, ..., J$, i.e., the U_j source symbols of Chunk j should be allocated equally among layers $h_{j-1}+1$ to h_j.

To prove this, assume there exists allocation Scheme C, where there is at least one chunk (index denoted by j) within which the U_j source symbols are not equally allocated among layers $h_{j-1}+1$ and h_j. Then, because K_j is a non-decreasing set and $\sum_{i=h_{j-1}+1}^{h_j} K_i = U_j$, we have, for allocation Scheme C, $K^C_{h_j} > U_j/(h_j - h_{j-1}) = U_j/l_j$, where $K^C_l, l = 1, 2, ..., L$ represents the source symbol to layer allocation for Scheme C. We next construct allocation Scheme D where $K^D_{h_{j-1}+1} = K^D_{h_{j-1}+2} = ... = K^D_{h_j} = U_j/l_j$ with all the other chunks allocated identically to Scheme C, where $K^D_l, l = 1, 2, ..., L$ represents the source symbols to layer allocation for Scheme D. Because $K^C_{h_j} > U_j/l_j = K^D_{h_j}$ and $K^C_{h_i} = K^D_{h_i}$ for all $i \neq j$, it is obvious from the cost function expression (6), $M^C_{av} = \sum_{i=1}^{J} \eta_i K^C_{h_i} > \sum_{i=1}^{J} \eta_i K^D_{h_i} = M^D_{av}$, where M^C_{av} and M^D_{av} are the overall costs of Scheme C and Scheme D, respectively. Hence, allocation Scheme C does not minimize the cost function. Therefore, an optimal solution that minimizes M_{av} should allocate U_j symbols among layers $h_{j-1}+1$ to h_j equally for $j = 1, 2, .., J$, i.e., $K_1 = K_2 = ... = K_{h_1} \leq K_{h_1+1} = ... = K_{h_2} \leq ... \leq K_{h_{J-1}+1} = ... = K_{h_J} = K_L$. **QED**

Therefore, we can reduce the number of parameters by using one parameter K_{h_j} to represent the $K_l, h_{j-1}+1 \leq l \leq h_j$ values of Chunk j, i.e., $K_{h_{j-1}+1} = K_{h_{j-1}+2} = ... = K_{h_j} = U_j/l_j$. Using the above, the cost function can be further simplified as

$$M_{av} = \sum_{j=1}^{J} \eta_j K_{h_j} = \sum_{j=1}^{J} \eta_j U_j/l_j = \sum_{j=1}^{J} \alpha_j/l_j, \tag{14}$$

where

$$\alpha_j \equiv \eta_j U_j = w_j U_j (1+\omega) \int_0^1 \frac{f_{\sigma_j}(x)}{1-x} dx. \tag{15}$$

Also, using the constraint that K_j is a non-decreasing set, Eq. (1), can be transformed to $U_1/l_1 \leq U_2/l_2 ... \leq U_J/l_J$.

C. Transformation to a convex optimization problem

In summary, the allocation problem described by Problem 1 can be transformed to the following equivalent problem,

Problem 2:

$$\min_{l_1,\dots,l_J} \quad \sum_{i=1}^{J} \alpha_i/l_i \tag{16}$$

subject to

$$\sum_{i=1}^{J} l_i = L \tag{17}$$

$$0 \le l_J/U_J \le l_{J-1}/U_{J-1} \dots \le l_1/U_1, \tag{18}$$

If we ignore the constraints that the $l_i, i = 1, 2, .., J$ are integers, we can show that Problems 2 is a convex optimization problem. This can be proven by showing that the objective function is convex and all the constraint functions are linear [11].

IV. Analytical solutions

We next show that the optimal solution of Problem 2 can be obtained analytically. As a first step, we solve Problem 2 without the inequality constraints provided by (18), which is easily done by using Lagrange's method. The Lagrange L function in this case would be

$$\begin{aligned} L(l,v) &= \sum_{i=1}^{J} \alpha_i/l_i + v(\sum_{i=1}^{J} l_i - L) \\ &= \sum_{i=1}^{J} (\alpha_i/l_i + v l_i) - vL. \end{aligned} \tag{19}$$

The optimal solutions l_i for Problem 2 without inequality constraints (18) minimize $\alpha_i/l_i + vl_i$ for all $i = 1, 2, ..., J$. By setting the derivative of $\alpha_i/l_i + vl_i$ with respect to l_i to zero, the optimal l_i can be obtained as

$$l_i = \frac{L\sqrt{\alpha_i}}{\sum_{i=1}^{J}\sqrt{\alpha_i}}, \qquad j = 1, 2, ..., J. \tag{20}$$

Now if the above solution satisfies the inequality constraints provided by (18), then we are done. In order to solve the problem when (20) does not satisfy inequality constraints (18), we deploy a similar method that used in [14]. We first prove the following fact:

Lemma 2: For any $j \in (1, 2, ..., J-1)$, if $\frac{\sqrt{\alpha_{j+1}}}{U_{j+1}} > \frac{\sqrt{\alpha_j}}{U_j}$, then the optimal solution of Problem 1 satisfies $\frac{l_j}{U_j} = \frac{l_{j+1}}{U_{j+1}}$.

Proof: Assume $\frac{\sqrt{\alpha_{j+1}}}{U_{j+1}} > \frac{\sqrt{\alpha_j}}{U_j}$, $j \in (1, 2, ..., J-1)$ and that there exists a set of optimal solutions $X = \{l_i^X, i = 1, 2, ..., J, \frac{l_j^X}{U_j} \neq \frac{l_{j+1}^X}{U_{j+1}}\}$. Now we create a new set of solutions $Y = \{l_i^Y, i = 1, 2, ..., J\}$ by perturbing l_j^X and l_{j+1}^X by a very small amount $\Delta l > 0$ such that the constraints (18) are still valid, i.e., for $i = 1, 2, .., J$,

$$l_i^Y = \begin{cases} l_i^X - \triangle\, l & \text{if } i = j \\ l_i^X + \triangle\, l & \text{if } i = j+1 \\ l_i^X & \text{if } i \neq j, j+1. \end{cases} \tag{21}$$

Now the difference in the cost functions due to (21) is

$$\begin{aligned} M_{av}^Y - M_{av}^X &= \tfrac{\alpha_j}{l_j^X - \Delta l} + \tfrac{\alpha_{j+1}}{l_{j+1}^X + \Delta l} - \tfrac{\alpha_j}{l_j^X} - \tfrac{\alpha_{j+1}}{l_{j+1}^X} \\ &= \Delta l \left(\tfrac{\alpha_j}{l_j^X (l_j^X - \Delta l)} - \tfrac{\alpha_{j+1}}{l_{j+1}^X (l_{j+1}^X + \Delta l)} \right). \end{aligned} \tag{22}$$

As $\frac{\sqrt{\alpha_{j+1}}}{U_{j+1}} > \frac{\sqrt{\alpha_j}}{U_j}$ and $\frac{l_j^X}{U_j} > \frac{l_{j+1}^X}{U_{j+1}}$, we have

$$\frac{\alpha_{j+1}}{\alpha_j} > \frac{U_{j+1}^2}{U_j^2} > \frac{(l_{j+1}^X)^2}{(l_j^X)^2}. \tag{23}$$

Therefore, if $\Delta l > 0$ is chosen to be small enough, then we certainly have $\frac{\alpha_{j+1}}{\alpha_j} > \frac{l_{j+1}^X (l_{j+1}^X + \Delta l)}{l_j^X (l_j^X - \Delta l)}$. Hence $M_{av}^Y - M_{av}^X < 0$, which contradicts the assumption that the set of solutions X is optimum. Therefore, the optimal solution must satisfy $\frac{l_j}{U_j} = \frac{l_{j+1}}{U_{j+1}}$ if $\frac{\sqrt{\alpha_{j+1}}}{U_{j+1}} > \frac{\sqrt{\alpha_j}}{U_j}$ for any $j = 1, 2, ..., J-1$. **QED**

With the above lemma, if $\frac{\sqrt{\alpha_{j+1}}}{U_{j+1}} > \frac{\sqrt{\alpha_j}}{U_j}$, we have $K_{h_j} = \frac{U_j}{l_j} = \frac{U_{j+1}}{l_{j+1}} = K_{h_{j+1}}$. Therefore, we can reduce the number of parameters further by grouping Chunk j and Chunk $j+1$ into a new chunk, i.e, $U_j^{new} = U_j + U_{j+1}$, $l_j^{new} = l_j + l_{j+1}$, $\alpha_j^{new} = (\alpha_j/l_j + \alpha_{j+1}/l_{j+1})(l_j + l_{j+1}) = (\alpha_j/U_j + \alpha_{j+1}/U_{j+1})(U_j + U_{j+1})$, $J_{new} = J - 1$ and decrease the indices of all parameters with index larger than $j+1$ by one. This procedure is repeated until no $j \in \{1, 2, ..., J-1\}$ with $\frac{\sqrt{\alpha_{j+1}}}{U_{j+1}} > \frac{\sqrt{\alpha_j}}{U_j}$ exists, i.e., $\frac{\sqrt{\alpha_{j+1}}}{U_{j+1}} \le \frac{\sqrt{\alpha_j}}{U_j}$ for all $j = 1, 2, ..., J-1$.

Denote $\hat{x}$ as the new value of variable x after the chunk grouping process, Problem 2 is then transformed to:

Problem 3:

$$\min_{\hat{l}_1,\dots,\hat{l}_{\hat{J}}} \quad \sum_{i=1}^{\hat{J}} \hat{\alpha}_i/\hat{l}_i \tag{24}$$

subject to

$$\sum_{i=1}^{\hat{J}} \hat{l}_i = L \tag{25}$$

$$0 \le \hat{l}_{\hat{J}}/\hat{U}_{\hat{J}} \le \hat{l}_{\hat{J}-1}/\hat{U}_{\hat{J}-1} \dots \le \hat{l}_1/\hat{U}_1, \tag{26}$$

where $\frac{\sqrt{\hat{\alpha}_j}}{\hat{U}_j}$ is a non-increasing set for $j = 1, 2, ..., \hat{J}$. Following the same approach as in (20), the optimal solution for Problem 3 without constraint (26) is:

$$\hat{l}_j^0 = \frac{L\sqrt{\hat{\alpha}_j}}{\sum_{i=1}^{\hat{J}}\sqrt{\hat{\alpha}_i}}, \qquad j = 1, 2, ..., \hat{J}. \tag{27}$$

Since $\frac{\hat{l}_j^0}{\hat{U}_j} = \frac{\sqrt{\hat{\alpha}_j}}{\hat{U}_j} \frac{L}{\sum_{i=1}^{\hat{J}}\sqrt{\hat{\alpha}_i}} \ge \frac{\sqrt{\hat{\alpha}_{j+1}}}{\hat{U}_{j+1}} \frac{L}{\sum_{i=1}^{\hat{J}}\sqrt{\hat{\alpha}_i}} = \frac{\hat{l}_{j+1}^0}{\hat{U}_{j+1}}$, the inequality constraints (26) are satisfied. Therefore, the solutions $\hat{l}_j^0, j = 1, 2, ..., \hat{J}$ given by (27) form an optimal set of solutions for Problem 3. We have thus found an analytical solution to Problem 2, which is equivalent to Problem 1.

Finally, assume that we have reached a set of optimal solutions l_i^0, $i = 1, 2, ..., J$ for Problem 2. We can proceed to find a close-to-optimal solution with integer constraints: $l_1 = \lfloor l_1^0 \rceil$ and $l_j = \lfloor \sum_{i=1}^{j} l_i^0 - \sum_{i=1}^{j-1} l_i \rceil$ for $j = 2, ..., J$, where $\lfloor x \rceil$ is the nearest integer to x. Each chunk of U_j source symbols is allocated nearly equally in the sense that there is maximum of one symbol difference among $K_{h_j-1}, \dots$ and K_{h_j}.

The analytical solution only requires the computation of J coefficients $\frac{\sqrt{\alpha_j}}{U_j}$, and perform at most $J-1$ chunk grouping processes, and compute J variables according to (27). In the worst case, the analytical solution requires roughly $14J$ elementary operations, which is linear in J and independent of L. The complexity of the analytical solution is significantly lower than that of the greedy search algorithm proposed in [9], which requires roughly $JL^{J-1}/2$ elementary operations. For example, if the packet size $L = 1000$, with $J = 4$ user classes, the greedy search algorithm requires on the order of 10^9 elementary operations, while the analytical solution requires less than 100 elementary operations.

V. Numerical results

To illustrate the above analytical method of solving Problem 1, two numerical examples are considered. For simplicity, all the channel erasure rates σ_j are assumed to be constants. For the first example, the same parameters as the simulation setup in [9] is used for verification purpose. In the setup, the server is multicasting a scalable image or video sequence to two user classes. The total number of layers in the packetization scheme is fixed at $L = 47$. The two user classes have a target QoS (PSNR) $\gamma_1 = 27$ dB and $\gamma_2 = 30$ dB, respectively. From the PSNR curve of the image, the minimum number of source symbols needed to provide the corresponding QoS requirements are $\lceil f^{-1}(\gamma_1)\rceil = 11072$ and $\lceil f^{-1}(\gamma_2)\rceil = 24728$, respectively [9]. Therefore the chunk sizes $U_1 = \lceil f^{-1}(\gamma_1)\rceil = 11072$ and $U_2 = \lceil f^{-1}(\gamma_2)\rceil - \lceil f^{-1}(\gamma_1)\rceil = 13656$. For each user class, the channel erasure rate is $\sigma_1 = \sigma_2 = 0.0549$. [2] The overhead of the rateless code is assumed to be $\omega = 5\%$. The weighting coefficient for Class 1 users w_1 can be varied to obtain different results.

In order to find the best allocation scheme based on the packetization structure, the following relevant coefficients are first computed. From (15), $\alpha_j = \eta_j U_j = \frac{U_j w_j (1+\omega)}{1-\sigma_j} = 1.111 U_j w_j$ for $j = 1, 2$ and the weighting coefficient $w_2 = 1 - w_1$. We now determine a sufficient condition for the optimal solution to satisfy $K_{h_1} = K_{h_2}$. According to Lemma 2, if $\frac{\sqrt{\alpha_2}}{U_2} > \frac{\sqrt{\alpha_1}}{U_1}$, the optimal solution of Problem 1 should satisfy $\frac{l_1}{U_1} = \frac{l_2}{U_2}$. The above condition is equivalent to $\frac{\sqrt{w_2 U_2 (1+\omega)/(1-\sigma_2)}}{U_2} > \frac{\sqrt{w_1 U_1 (1+\omega)/(1-\sigma_1)}}{U_1}$. Since $\sigma_1 = \sigma_2$ and $w_2 = 1 - w_1$, this condition is equivalent to $\frac{1-w_1}{w_1} > \frac{U_2}{U_1}$ or $w_1 < 0.448$. Therefore, for $w_1 < 0.448$, the optimal solution is to allocate the number of source symbols equally among the $L = 47$ layers, i.e., the optimal allocation scheme is an equal error protection (EEP) scheme. Hence if $w_1 < 0.448$, each layer should have $\lceil f^{-1}(\gamma_2)\rceil / L = 24728/47 = 526.13$ source symbols, To satisfy the integer constraints, we should then allocate 526 source symbols for the first $47 \times 527 - 24728 = 41$ layers and allocate 527 symbols for the last 6 layers. From the above process, we also discover the following interesting fact as a special case of Lemma 2: in Problem 1, if there are only two user classes with the same channel conditions and the weighting coefficient of class 1 users satisfies $w_1 < U_1/(U_1 + U_2)$, then the optimal allocation scheme is an EEP scheme.

[2]The channel erasure rates are chosen such that they are equivalent to the parameterizations used in [9] in the sense that they provide the same channel quality factors δ_j defined by (7).

Now if $w_1 > 0.448$, the problem is transformed to Problem 3 and the optimal solution is given by (27), i.e., $l_1^0 = \frac{L\sqrt{\alpha_1}}{\sum_{i=1}^{2}\sqrt{\alpha_i}} = L\frac{\sqrt{w_1 U_1}}{\sqrt{w_1 U_1} + \sqrt{w_2 U_2}}$. Therefore, if $w_1 = 0.6$, then $l_1^0 = 24.65$. With integer constraints, we have $l_1 = \lfloor l_1^0 \rceil = 25$, $l_2 = 22$. To allocate $U_1 = 11072$ symbols nearly equally among the first 25 layers, we obtain $K_1 = K_2 = K_3 = 442$ and $K_4 = K_5 = \dots = K_{25} = 443$. Similarly to allocate $U_2 = 13656$ symbols among the remaining 22 layers, we obtain $K_{26} = K_{27} = \dots = K_{31} = 620$ and $K_{32} = K_{33} = \dots = K_{47} = 621$. If $w_1 = 0.8$, then $l_1^0 = L\frac{\sqrt{w_1 U_1}}{\sqrt{w_1 U_1} + \sqrt{w_2 U_2}} = 30.22$ and $l_1 = \lfloor l_1^0 \rceil = 30$. If $w_1 = 1$, then $l_1 = L = 47$. Therefore, the optimal solution is to allocate $U_1 = 11072$ symbols among all 47 layers. The number of source symbols allocated for each layer when $w_1 = 0.4, 0.6, 0.8$ and 1 are shown in Fig. 2. By comparing Fig. 2 to Fig. 2 of [9] which was obtained by the greedy search method, we can see that all the results match each other accurately. Fig. 3 presents the corresponding cost M_{av} (6) of the optimal allocation scheme obtained using the analytical solution in comparison with that of the EEP allocation scheme for different choices of w_1. It can be seen that the advantage of using the optimal allocation scheme over the EEP allocation scheme is significant when $w_1 > 0.5$. Readers can refer to [9] for comparisons of the rateless-UEP scheme used in this paper to the UEP scheme in [10], where the advantages of rateless codes over fixed-rate RS codes are clearly demonstrated.

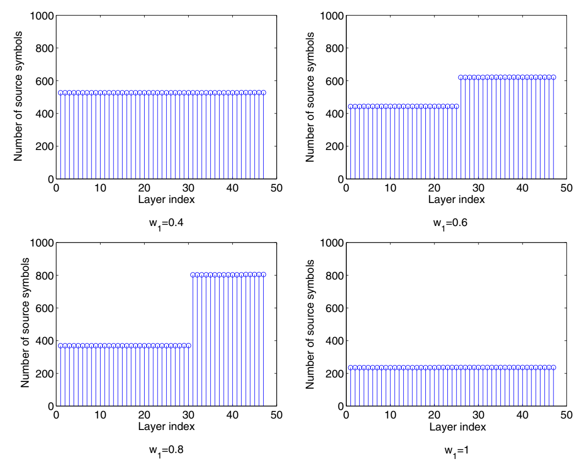

Fig. 2. Allocation of source symbols for an asynchronous multimedia multicast system with two user classes with different values of w_1.

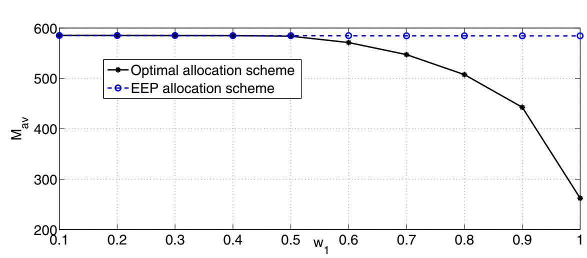

Fig. 3. Comparisons between the average cost M_{av} of the optimal UEP allocation scheme and that of the EEP allocation scheme with different values of w_1.

To demonstrate the solution of Problem 1 with more than

TABLE I
PARAMETERS OF THE ASYNCHRONOUS MULTIMEDIA MULTICAST EXAMPLE WITH FOUR USER CLASSES

User class index (j)	1	2	3	4
User erasure rate σ_j	0.6	0.5	0.4	0
PSNR threshold γ_j(dB)	25.79	27.25	29	40.28
$f^{-1}(\gamma_j)$	400	700	1155	3800
Weighting coefficient (w_j)	0.4	0.1	0.3	0.2
Chunk size U_j	400	300	455	2645
$\alpha_j = \frac{w_j U_j (1+\omega)}{1-\sigma_j}$	420.0	63.00	238.9	555.5
$\frac{\sqrt{\alpha_j}}{U_j}$	0.0512	0.0265	0.0340	0.0089

TABLE II
PARAMETERS AND THE OPTIMAL SOLUTION AFTER TRANSFORMATION

User class index (j)	1	2	3
number of symbols for each chunk ($\hat{U_j}$)	400	755	2645
$\hat{\alpha_j}$	420.0	554.9	555.5
optimal solution $\hat{l_0}$	14.2	16.4	16.4

two classes, we consider an example where the server is multicasting a scalable video stream to four user classes. The number of symbols and the PSNR value of the video sequence are taken from Table I of [7]. The video sequence consists of one base layer (BL) and fourteen enhancement layers (EL) with a total of 3800 symbols where each symbol represents 50 bytes. The channel erasure rates σ_j, the PSNR threshold γ_j and the weighting coefficient w_j of each user class is shown in Table I . The total number of layers $L = 47$ and the rateless code's overhead is assumed to be $\omega = 0.05$. The number of symbols required to reach the PSNR threshold $f^{-1}(\gamma_j)$, U_j, $\alpha_j = w_j U_j (1+w)/(1-\sigma_j)$ and $\sqrt{\alpha_j}/U_j$ are computed based on the above given parameters.

Following the steps described in Section IV, we first compare the values of $\frac{\sqrt{\alpha_j}}{U_j}$ among different classes. Since $\frac{\sqrt{\alpha_3}}{U_3} > \frac{\sqrt{\alpha_2}}{U_2}$, from Lemma 2, the optimal solution should satisfy $\frac{l_2}{U_2} = \frac{l_3}{U_3}$. Therefore, this problem can be transformed to the form of Problem 3 by merging Chunk 2 and Chunk 3 to a new chunk with parameters $\hat{U_2} = U_2 + U_3 = 755$ and $\hat{\alpha_2} = (\alpha_j/U_j + \alpha_{j+1}/U_{j+1})(U_j + U_{j+1}) = 554.9$. Since $\frac{\sqrt{\hat{\alpha_j}}}{\hat{U_j}}$ is now non-increasing, the optimal solution of Problem 3 can be computed as $\hat{l}_j^0 = \frac{L\sqrt{\hat{\alpha}_j}}{\sum_{i=1}^{j}\sqrt{\hat{\alpha}_i}}, j = 1, 2, 3$. The parameters and solutions for the transformed problem, Problem 3, of this example are summarized in Table II. The optimal solution for Problem 2 can thus be obtained by $l_1^0 = \hat{l}_1^0$, $l_2^0 = \hat{l}_2^0 \frac{U_2}{U_2+U_3}$, $l_3^0 = \hat{l}_2^0 \frac{U_3}{U_2+U_3}$ and $l_4^0 = \hat{l}_4^0$. With integer constraints, we obtain $l_1 = 14$, $l_2 = 7$, $l_3 = 10$, $l_4 = 16$. Alternatively, we solve Problem 2 with parameters from Table I using CVX [12], a convex optimization software package for Matlab, and obtained the same optimal solution. The number of source symbols allocated to each layer, $K_i, i = 1, 2, ..., L$, are presented in Fig. 4.

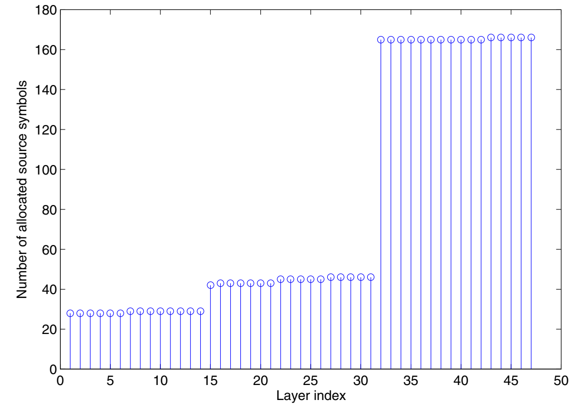

Fig. 4. Allocation of source symbols for the asynchronous multimedia multicast system with four user classes.

VI. CONCLUSIONS

We show that the problem of optimal allocation of UEP rateless codes for asynchronous multimedia multicast [9] can be transformed into a convex optimization problem when integer constraints are relaxed. The total number of variables for the transformed problem is reduced to be equal to the number of user classes. In addition, an analytical solution is found which has several orders of magnitude lower computational complexity compared to the greedy search algorithm proposed in [9]. Numerical results demonstrate that the optimal analytical solution matches the results of [9] as well as results obtained using convex optimization software, CVX [12].

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