This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

Minimum BER Transmit Optimization for Two-Input Multiple-Output Spatial Multiplexing

Neng Wang and Steven D. Blostein
Department of Electrical and Computer Engineering
Queen's University, Kingston, Ontario, Canada K7L 3N6
E-mail: nwang@ee.queensu.ca, steven.blostein@queensu.ca

Abstract— **A two-input multiple-output (TIMO) system represents an important special case of multiple-input multiple-output (MIMO) systems and occurs in practical scenarios where there are limitations on cost and/or space to install more antennas, in MIMO with transmit antenna selection which selects two out of multiple transmit antennas and turns MIMO into TIMO, or in cooperative communications with two single-antenna mobiles sharing their antennas. In this paper, minimum bit error rate (MBER) transmit optimization for TIMO spatial multiplexing systems is investigated. Approximate MBER transmit power allocation for a variety of receiver structures is proposed. Transmit beamforming schemes using 4-ary Pulse-Amplitude-Modulation (4-PAM) and Quaternary Phase-Shift-Keying (QPSK) pre-mixing are also proposed, which eliminate error floors in ill-conditioned TIMO channels. It is shown both analytically and by numerical simulations that the proposed schemes offer superior performance over existing schemes. Essentially, the proposed transmit optimization provides a simple and efficient means to utilize partial channel state information at the transmitter.**

I. Introduction

Wireless communications using multiple transmit and receive antennas, known as multiple-input multiple-output (MIMO) systems, offers key advantages over single-input single-output (SISO) systems, such as diversity and spatial multiplexing gains [1]. Our goal of this paper is to investigate transmit optimization for a MIMO spatial multiplexing system with two transmit antennas, known also as two-input multiple-output (TIMO). The study of such a system can be motivated in a number of ways: 1) TIMO systems are important in practical scenarios where there are limitations on cost and/or space to install more antennas; 2) a virtual TIMO channel is created when two single-antenna mobiles operate in cooperative communication mode [2]; 3) when transmit antenna selection is employed in MIMO to achieve diversity with reduced cost of transmit radio frequency chains [3], selecting two out of multiple transmit antennas turns MIMO into TIMO; 4) it is easier to analyze TIMO systems than the more general MIMO systems, and these analyses offer insights into MIMO system design and performance.

When channel state information (CSI) is available at the transmitter, system performance can be improved. Transmit optimization is receiver-dependent. Signal reception for spatial multiplexing can employ criteria such as maximum likelihood (ML), zero-forcing (ZF), minimum mean squared-error (MMSE), successive interference cancellation (SIC), or ordered SIC (OSIC) as, for example, in the case of the Vertical Bell Laboratories Layered Space-Time (V-BLAST) [4]. Efforts to optimize MIMO transceiver structures have involved linear MMSE precoding/decoding [5], minimum bit error rate (MBER) precoding for ZF equalization (ZF-MBER) [6] and limited feedback precoding schemes [7]. These schemes, however, generally require high feedback overhead and/or high complexity processing, e.g., matrix transformations at both the transmitter and the receiver.

In this paper, we consider simplified precoding by introducing structural constraints to precoding. Minimization of bit error rate (MBER) is employed as the optimization criterion. We categorize TIMO channels into well- and ill-conditioned cases. Channel condition is determined by a number of factors, such as Ricean factor and/or spatial correlation. For well-conditioned TIMO channels, precoding is constrained to *transmit power allocation*, i.e., we optimize only the transmitted power of signal streams, resulting in reduced processing complexity compared to general precoding. In principle, power allocation for general MIMO systems proposed in [8] can be applied directly to TIMO systems. Nevertheless, useful simplifications and insights can be obtained for TIMO systems, which are practical in their own right and do not apply to general MIMO systems. An example is the ill-conditioned pinhole channel that can be easily modelled in TIMO systems. When the TIMO channel is ill-conditioned, both ZF-MBER and MMSE precoding and the proposed power allocation are shown to experience error floors. We introduce added degrees of freedom and develop *transmit beamforming* schemes under a MBER criterion, which can eliminate these error floors. Compared to precoding which exploits spatial correlation of transmit antennas [9], the proposed transmit beamforming method utilizes instantaneous CSI and does not depend on the channel model used.

II. TIMO Channel and Transceiver

Consider a TIMO system with $N_r \geq 2$ receive antennas. The received signal can be modelled as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \boldsymbol{\eta} = s_1\mathbf{h}_1 + s_2\mathbf{h}_2 + \boldsymbol{\eta}, \qquad (1)$$

where $\mathbf{s} = [s_1, s_2]^T$ denotes a transmitted signal vector, $\mathbf{H} = [\mathbf{h}_1 \vdots \mathbf{h}_2]$ is an $N_r \times 2$ channel matrix, which is assumed to be general correlated Ricean fading [10], and $\boldsymbol{\eta}$ is an $N_r \times 1$

additive Gaussian noise vector. For simplification of analysis purposes, we assume white noise and input, i.e., $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = E_s\mathbf{I}_2$ and $\mathbb{E}[\boldsymbol{\eta}\boldsymbol{\eta}^H] = N_0\mathbf{I}_{N_r}$, and define the signal-to-noise ratio (SNR) $\gamma_s \overset{\text{def}}{=} E_s/N_0$. Binary Phase Shift Keying (BPSK) modulation is assumed. Extension to other constellations will be addressed later.

A. TIMO Signal Reception

1) ZF Receiver: With ZF equalization, the transmitted signal is estimated as $\hat{\mathbf{s}} = \mathbf{H}^\dagger\mathbf{r} = \mathbf{s} + \mathbf{H}^\dagger\boldsymbol{\eta}$. The noise covariance per real and imaginary dimension in $\hat{\mathbf{s}}$ can be calculated to be $\frac{N_0}{2}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}$. The decision-point SNR of the k-th signal stream is obtained as

$$\gamma_{Z,k} = \frac{2E_s}{N_0\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{k,k}} \overset{\text{def}}{=} 2\gamma_s g_{Z,k}^2, \quad k = 1, 2, \tag{2}$$

where $g_{Z,k}^2 \overset{\text{def}}{=} [(\mathbf{H}^H\mathbf{H})^{-1}]_{k,k}^{-1}$ denotes the power gain of k-th stream, and can be calculated as

$$g_{Z,1}^2 = \frac{\Delta_\mathbf{H}}{\|\mathbf{h}_2\|^2}, \qquad g_{Z,2}^2 = \frac{\Delta_\mathbf{H}}{\|\mathbf{h}_1\|^2}, \tag{3}$$

where $\Delta_\mathbf{H} \overset{\text{def}}{=} \|\mathbf{h}_1\|^2\|\mathbf{h}_2\|^2 - |\mathbf{h}_2^H\mathbf{h}_1|^2$.

2) SIC Receiver: Without loss of generality (w.o.l.g.), we assume that stream $k = 1$ is detected first. Assuming ZF equalization is employed, the power gain in detecting s_1 is given by

$$g_{S,1}^2 = g_{Z,1}^2 = \frac{\Delta_\mathbf{H}}{\|\mathbf{h}_2\|^2}. \tag{4}$$

Assuming that $\hat{s}_1 = s_1$, the interference due to the first stream is then regenerated and subtracted, i.e., $\mathbf{r}' = \mathbf{r} - \hat{s}_1\mathbf{h}_1 = s_2\mathbf{h}_2 + \boldsymbol{\eta}$. The detection of s_2 in $\mathbf{r}'$ with ZF equalization is given by $\hat{s}_2 = \mathbf{h}_2^\dagger\mathbf{r}' = s_2 + \frac{\mathbf{h}_2^H\boldsymbol{\eta}}{\|\mathbf{h}_2\|^2}$, with power gain

$$g_{S,2}^2 = \|\mathbf{h}_2\|^2. \tag{5}$$

3) OSIC Receiver: To improve SIC performance, the streams can be reordered based on SNR at each stage. The SNR-based ordering scheme [4] detects the stream with largest decision-point SNR first. From (2) and (3), the stream to be detected first is

$$k_1 = \arg\max_{l\in\{1,2\}} \gamma_{Z,l} = \arg\max_{l\in\{1,2\}} g_{Z,l}^2 = \arg\max_{l\in\{1,2\}} \|\mathbf{h}_l\|^2, \tag{6}$$

i.e., SNR-based ordering is equivalent to norm-based ordering in TIMO systems. Therefore, we obtain the power gains as

$$g_{O,1}^2 = \frac{\Delta_\mathbf{H}}{\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\}}, \quad g_{O,2}^2 = \min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\}. \tag{7}$$

The average BER of the above receivers can be calculated as[1]

$$\bar{P}(\gamma_s; g_1^2, g_2^2) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2}\right), \tag{8}$$

where $\mathcal{Q}(x) \overset{\text{def}}{=} \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-y^2/2}dy$. The gains g_k^2's depend on the receiver structure and are given in (3), (4), (5) and (7).

B. Ill-Conditioned TIMO Channels

Since the $\mathcal{Q}(\cdot)$ function decreases rapidly in its argument, the average BER in (8) is dominated by the term with smaller power gain. In the extreme case with vanishing power gain, the system experiences an error floor. We refer to this as an *ill-conditioned* TIMO channel. From gains in (3), (4), (5) and (7), the channel is ill-conditioned when either $\Delta_\mathbf{H} \cong 0$ or $\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\} \cong 0$.

- The condition $\Delta_\mathbf{H} \cong 0$ is equivalent to $\|\mathbf{h}_1\|^2 \cdot \|\mathbf{h}_2\|^2 \cong |\mathbf{h}_1^H\mathbf{h}_2|^2$. We have $\frac{|\mathbf{h}_1^H\mathbf{h}_2|^2}{\|\mathbf{h}_2\|^2} = \mathbf{h}_1^H\boldsymbol{\Upsilon}_{\mathbf{h}_2}\mathbf{h}_1 = \|\boldsymbol{\Upsilon}_{\mathbf{h}_2}\mathbf{h}_1\|^2 \cong \|\mathbf{h}_1\|^2$, where $\boldsymbol{\Upsilon}_\mathbf{X} \overset{\text{def}}{=} \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H$, is the projection matrix. Therefore, w.o.l.g., we can assume $\mathbf{h}_2 \cong a\cdot\mathbf{h}_1$ with $a \in \mathbb{C}$, and the channel matrix

$$\mathbf{H} \cong \mathbf{h}_1[1 \ \ a], \tag{9}$$

  which is also an example of a "pinhole" channel [11]. The least-squares (LS) estimate of a can be found to be

$$\hat{a}_{LS} = \mathbf{h}_1^\dagger\mathbf{h}_2 = \frac{\mathbf{h}_1^H\mathbf{h}_2}{\|\mathbf{h}_1\|^2}. \tag{10}$$

- It can be shown that when $\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\} \cong 0$, the model (9) is still valid with $a \cong 0$.

Furthermore, it can be shown that the model (9) is valid for ill-conditioned TIMO with linear ZF-MBER and MMSE precoding/decoding as well.

III. Transmit Power Allocation for TIMO

Denote the power allocated to the k-th stream as p_k^2 ($k = 1, 2$). The received signal is given by $\mathbf{r} = p_1 s_1\mathbf{h}_1 + p_2 s_2\mathbf{h}_2 + \boldsymbol{\eta}$. The transmit power is normalized as $p_1^2 + p_2^2 = 2$.

A. Approximate MBER (AMBER) Power Allocation

The average BER of TIMO with power allocation can be obtained by generalizing (8) to

$$\bar{P}(\gamma_s; \{g_k^2\}; \{p_k^2\}) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2 p_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2 p_2^2}\right). \tag{11}$$

To minimize (11) under transmit power constraint, no closed-form solution exists. However, taking the approach in [12], we approximate the objective function to obtain a closed-form AMBER solution with performance very close to the MBER solution. Using the approximate BER formula given in [13][2], an AMBER power allocation is obtained as [12]

$$p_k^2 = \gamma_s^{-1}g_k^{-2}\left(\ln g_k^2 + \nu\right)_+, \quad k = 1, 2, \tag{12}$$

where $(x)_+ \overset{\text{def}}{=} \max\{0, x\}$, and ν is chosen to satisfy power constraint. Note the fact that the total transmit power $p_1^2 + p_2^2$ is a piecewise-linear function in ν, with breakpoints at $-\ln g_1^2$

[1]For SIC and OSIC receivers, (8) is only a lower bound due to the neglecting of error propagation. However, at moderate-to-high SNR regimes, this lower bound closely approximates the average BER since error propagation is minimal. The simulation results in Section V, however, take error propagation into account.

[2]The BER can approximated as $P_b(\gamma) \approx \frac{1}{5}\exp\{-c\gamma\}$, where c is a constellation-specific constant [13]. For BPSK modulation, $c = 1$.

 0-7803-9415-1/05/$20.00

and $-\ln g_2^2$. W.o.l.g., we assume $g_1^2 \geq g_2^2$. We can simplify the solution (12) as

$$\begin{cases} p_1^2 = 2, \;\; p_2^2 = 0 & \text{if } \ln\left(\frac{g_1^2}{g_2^2}\right) \geq 2\gamma_s g_1^2 \\ p_k^2 = \frac{\ln g_k^2 + \nu_a}{\gamma_s g_k^2}, (k=1,2) & \text{otherwise} \end{cases}, \quad (13)$$

where $\nu_a = \frac{2\gamma_s g_1^2 g_2^2 - g_1^2 \ln g_2^2 - g_2^2 \ln g_1^2}{g_1^2 + g_2^2}$.

In the first case of (13), the solution $p_1^2 = 2$ and $p_2^2 = 0$ implies that the stream with weaker power gain is dropped. This occurs when $\ln g_1^2 - \ln g_2^2 \geq 2\gamma_s g_1^2$, i.e., either $g_1^2 \gg g_2^2$ or γ_s is small. In this case, if both streams are used for transmission, an error floor is inevitable. To avoid such error floors, only one stream may be transmitted, and power allocation turns out to be *transmit antenna selection*. We note that the condition $\ln g_1^2 - \ln g_2^2 \geq 2\gamma_s g_1^2$ is also a necessary condition of the ill-conditioned TIMO channel defined in Section II-B. Alternative schemes to eliminate error floor effects will be addressed in Section IV.

B. Remarks

1) Feedback Overhead and Complexity: Using a general precoding method, either the channel or precoding matrix is required at the transmitter. The proposed power allocation scheme requires only transmitted power information. Precoding schemes require diagonalization of a channel matrix [5]–[7]. Using power allocation, operations performed at the transmitter are trivial.

2) Extension to General MIMO and Modulations: Although our discussion of power allocation is focused on TIMO channels and BPSK modulation, extension to general MIMO systems and modulations is straightforward [8].

3) Application Scenarios: A TIMO configuration is appropriate for the uplink of a wireless system, where each mobile terminal is equipped with dual transmit antenna while the basestation has more antennas. The downlink, on the other hand, has a multiple-input two-output (MITO) structure. To exploit the inherent transmit diversity in such a MITO system, transmit processing is necessary [3], [5], [6].

4) Asymptotic Performance: At moderate-to-high SNR ($\gamma_s \gg 1$), the BER of power allocation method (11) can be approximated as

$$\tilde{P}\left(\gamma_s; g_1^2, g_2^2\right) \cong \frac{1}{10}\left(g_1^{-2} + g_2^{-2}\right) \exp\left\{-\frac{2\gamma_s}{g_1^{-2} + g_2^{-2}}\right\}. \quad (14)$$

5) Effects of Power Allocation on Detection Ordering: For OSIC, we examine the effect of power allocation on ordering. W.o.l.g., we assume $\|\mathbf{h}_1\|^2 \geq \|\mathbf{h}_2\|^2$. In norm-based ordering, s_1 is detected first. Denote the corresponding power gains as, respectively, $\alpha_1^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_2\|^2}$, $\alpha_2^2 = \|\mathbf{h}_2\|^2$. The opposite detection ordering has power gains denoted as, respectively, $\beta_1^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_1\|^2}$, $\beta_2^2 = \|\mathbf{h}_1\|^2$. While a general analysis is difficult, we consider the asymptotic case. From (14), $\tilde{P}_a\left(\gamma_s; g_1^2, g_2^2\right)$ is an monotone increasing function in $g_1^{-2} + g_2^{-2}$. Note that $\alpha_1^2\alpha_2^2 = \beta_1^2\beta_2^2$. By the above assumption ($\|\mathbf{h}_1\|^2 \geq \|\mathbf{h}_2\|^2$), we have $\beta_1^2 \leq \alpha_1^2 \leq \beta_2^2$, and $\beta_1^2 \leq \alpha_2^2 \leq \beta_2^2$. Therefore, $\alpha_1^{-2} + \alpha_2^{-2} \leq \beta_1^{-2} + \beta_2^{-2}$. We conclude

$$\tilde{P}_a(\gamma_s; \alpha_1^2, \alpha_2^2) \leq \tilde{P}_a(\gamma_s; \beta_1^2, \beta_2^2),$$

i.e., norm-based ordering (6) is also asymptotically optimal for power allocation, in the sense of MBER.

6) Performance in Ill-Conditioned Channels: W.o.l.g., we assume $|a| \leq 1$ in the ill-conditioned channel (9). The power gains of OSIC can be obtained as $g_{O,1}^2 \cong 0$ and $g_{O,2}^2 = |a|^2\|\mathbf{h}_1\|^2$. Applying power allocation in (12), we obtain $p_1^2 = 0$, and $p_2^2 = 2$. The average BER of power allocation for ill-conditioned channels can be approximated as $\bar{P}(\gamma_s; \mathbf{h}_1, a) \cong \frac{1}{10} + \frac{1}{10}\exp\left\{-4\gamma_s|a|^2\|\mathbf{h}_1\|^2\right\}$, which experiences an obvious error floor. This motivates our study of transmit beamforming for ill-conditioned TIMO channels.

IV. Transmit Beamforming for Ill-Conditioned TIMO Channels

W.o.l.g., we assume $\|\mathbf{h}_1\|^2 \geq \|\mathbf{h}_2\|^2$, or, equivalently, $|a| \leq 1$. BPSK modulation is assumed. Extension to high-order modulations is addressed in [14].

A. Signal Reception and Performance

Consider the received signal (1) in an ill-conditioned TIMO channel (9), for which we have

$$\mathbf{H}^\dagger = \left(1 + |a|^2\right)^{-1} \|\mathbf{h}_1\|^{-2}\mathbf{H}^H.$$

The ZF equalization output can be obtained as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{H}^\dagger \mathbf{r} = \frac{1}{1+|a|^2}\left(\begin{bmatrix} s_1 + a s_2 \\ a^* s_1 + |a|^2 s_2 \end{bmatrix} + \frac{\mathbf{h}_1^H \boldsymbol{\eta}}{\|\mathbf{h}_1\|^2}\begin{bmatrix} 1 \\ a^* \end{bmatrix}\right). \quad (15)$$

From (15), we observe that the transmitted signals s_1 and s_2 are coupled in y_1 and y_2, and $y_2 = a^* y_1$. Therefore, it suffices to process one of the estimates, say y_1. Consider detecting s_1 and s_2 in a SIC fashion. Since the power of s_1 contained in y_1, $\mathbb{E}[|s_1|^2] = E_s$, is larger than that of s_2 contained in y_1, $\mathbb{E}[|a s_2|^2] = |a|^2 E_s$, the optimal order is to detect s_1 first. Denote $a_{\Re} = \Re\{a\}$. The BER can be calculated similarly as in OSIC receivers, i.e., $\bar{P}_{\mathrm{IC}}(\gamma_s; a, \mathbf{h}_1) \cong \frac{1}{4}\mathcal{Q}\left(\sqrt{2\gamma_s\|\mathbf{h}_1\|^2(1-a_{\Re})^2}\right) + \frac{1}{4}\mathcal{Q}\left(\sqrt{2\gamma_s\|\mathbf{h}_1\|^2(1+a_{\Re})^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s\|\mathbf{h}_1\|^2 a_{\Re}^2}\right)$, which experiences an error floor when either $a_{\Re} \cong 0$ or $a_{\Re} \cong \pm 1$. This occurs, e.g., when transmit fading coefficients are either in-phase or quadrature, respectively. Furthermore, it can be shown that power allocation cannot eliminate error floors in ill-conditioned channels. This motivates our study of a precoding scheme.

B. Transmit Beamforming Method

Consider general precoding for ill-conditioned TIMO channels. Denote the precoding matrix $\mathbf{P} \in \mathbb{C}^{2\times 2}$, with entries satisfy the normalized power constraint, $|p_{11}|^2 + |p_{12}|^2 +$

$|p_{21}|^2 + |p_{22}|^2 = 2$. The received signal is $\mathbf{r} = \mathbf{HPs} + \boldsymbol{\eta}$. With ZF equalization, the estimate of $\mathbf{s}$ is given by

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \frac{1}{1+|a|^2}\left(\begin{bmatrix} 1 & a \\ a^* & |a|^2 \end{bmatrix}\begin{bmatrix} p_{11}s_1 + p_{12}s_2 \\ p_{21}s_1 + p_{22}s_2 \end{bmatrix} + \frac{\mathbf{h}_1^H \boldsymbol{\eta}}{\|\mathbf{h}_1\|^2}\begin{bmatrix} 1 \\ a^* \end{bmatrix}\right). \quad (16)$$

From (16), we know $z_2 = a^* z_1$. Therefore, it suffices to process z_1 only. Consider

$$z_1' \stackrel{\text{def}}{=} (1+|a|^2)z_1 = (p_{11}+ap_{21})s_1 + (p_{12}+ap_{22})s_2 + \frac{\mathbf{h}_1^H \boldsymbol{\eta}}{\|\mathbf{h}_1\|^2}. \quad (17)$$

Signal detection can be performed in either one-dimensional (1D) or two-dimensional (2D) signal space.

1) 1D Signal Detection: W.o.l.g., we assume $\Re(p_{11} + ap_{21}) \geq 0, \Re(p_{12} + ap_{22}) \geq 0, \Re(p_{11} + ap_{21}) \geq \Re(p_{12} + ap_{22})$. As a result, s_1 is detected first. At moderate-to-high SNR, $\gamma_s \gg 1$, the average BER can be approximated as

$$\bar{P}_{\text{1D}}(\gamma_s; \mathbf{h}_1, a; \mathbf{P}) \cong \frac{1}{20} e^{-\gamma_s \|\mathbf{h}_1\|^2 [\Re(p_{11}+ap_{21}-p_{12}-ap_{22})]^2} + \frac{1}{10} e^{-\gamma_s \|\mathbf{h}_1\|^2 [\Re(p_{12}+ap_{22})]^2} \quad (18)$$

An approximate solution that minimizes (18) under the transmit power constraint is found to be [14]

$$\mathbf{P}_{\text{1D}} = \underbrace{(1+|a|^2)^{-1/2}\begin{bmatrix} 1 \\ a^* \end{bmatrix}}_{\mathbf{v}_{\text{BF}}} \times \underbrace{\sqrt{\frac{2}{5}}\begin{bmatrix} 2 & 1 \end{bmatrix}}_{\mathbf{v}_{\text{PM1}}^T} \quad (19)$$

The precoder (19) has rank one, and can be viewed as pre-mixing $\mathbf{v}_{\text{PM1}}$ followed by transmit beamforming $\mathbf{v}_{\text{BF}}$ pointing in the approximate MBER direction. Note that $\mathbf{v}_{\text{PM1}}$ pre-mixes two BPSK streams with different allocated powers into a 4-PAM stream. We refer to this scheme as *4-PAM beamforming*. Substituting (19) into (18), we obtain the approximate average BER as

$$\bar{P}_{\text{4-PAM}}^{\text{BF}}(\gamma_s; \mathbf{h}_1, a) \cong \frac{3}{20}\exp\left\{-\frac{2}{5}\gamma_s \|\mathbf{h}_1\|^2 (1+|a|^2)\right\}, \quad (20)$$

which does not experience error floors.

2) 2D Signal Detection: W.o.l.g., we assume s_1 and s_2 are, respectively, in-phase and quadrature components of (17). It can be shown that we can assume, w.o.l.g., $p_{11} \geq 0$, $jp_{12} \geq 0$, $ap_{21} \geq 0$, $jap_{22} \geq 0$. Accordingly, the average BER can be calculated as

$$\bar{P}_{\text{2D}}(\gamma_s; \mathbf{h}_1, a; \mathbf{P}) = \frac{1}{2}\mathcal{Q}\left(\sqrt{-2\gamma_s \|\mathbf{h}_1\|^2 (p_{12}+ap_{22})^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s \|\mathbf{h}_1\|^2 (p_{11}+ap_{21})^2}\right). \quad (21)$$

By using Lagrange multipliers, the exact MBER solution that minimizes (21) under the transmit power constraint is obtained as $p_{11} = (1+|a|^2)^{-1/2}$, $p_{12} = jp_{11}$, $p_{21} = a^* p_{11}$, $p_{22} = ja^* p_{11}$. Therefore, the precoder is given by

$$\mathbf{P}_{\text{2D}} = \underbrace{(1+|a|^2)^{-1/2}\begin{bmatrix} 1 \\ a^* \end{bmatrix}}_{\mathbf{v}_{\text{BF}}} \times \underbrace{\begin{bmatrix} 1 & j \end{bmatrix}}_{\mathbf{v}_{\text{PM2}}^T} \quad (22)$$

The rank-one precoder (22) can be viewed as pre-mixing, $\mathbf{v}_{\text{PM2}}$, followed by transmit beamforming, $\mathbf{v}_{\text{BF}}$, pointing in the MBER direction. Note that $\mathbf{v}_{\text{PM2}}$ pre-mixes two BPSK streams into a QPSK stream. We refer to this scheme as *QPSK beamforming*. Substituting (22) into (21), we obtain the average BER as

$$\bar{P}_{\text{QPSK}}^{\text{BF}}(\gamma_s; \mathbf{h}_1, a) = \mathcal{Q}\left(\sqrt{2\gamma_s \|\mathbf{h}_1\|^2 (1+|a|^2)}\right) \cong \frac{1}{5}\exp\left\{-\gamma_s \|\mathbf{h}_1\|^2 (1+|a|^2)\right\}, \quad (23)$$

which does not experience error floors. Comparing (23) with the performance of 4-PAM beamforming (20), we observe an approximate SNR gain of 2.5 ($\cong$ 4 dB), at the expense of increased detection complexity. This will be verified by simulations in Section V.

Remarks:

- *Feedback Overhead and Complexity:* From (19) and (22), only an estimate of a is required at the transmitter, which can be obtained using (10). Operations performed at the transmitter are also trivial.
- *Connection to ML-MD Precoding:* Both ML-MD precoding [15] and QPSK beamforming (22) pre-mix two BPSK streams into a QPSK stream. Minimum distance precoding applies to ML detection using a Euclidean distance criterion, while QPSK beamforming applies to ZF receivers under a MBER criterion. When the TIMO channel has rank one, these two methods coincide.

V. Numerical Results and Discussions

In our simulations, we adopt the spatial fading correlation model for general non-isotropic scattering given in [10]. The following parameters are chosen: $N_r = 4$ receive antennas; transmit and receive antenna spacings expressed in wavelengths are 0.5 and 10, respectively; angles of arrival/departure of the deterministic component are $\pi/6$ and 0, respectively; angle spread $10°$; and BPSK modulation is used for the purposes of comparison with [6].

Fig. 1 is a plot of the average BER for a variety of transceivers in an uncorrelated Rayleigh fading channel. To clarify the plot, performances of ZF with power allocation and SIC without power allocation are not shown since they are nearly identical to that of MMSE precoding/decoding; OSIC without power allocation (also not shown) has performance close to that of ZF-MBER precoding. We observe that at a BER of 10^{-3}, the proposed power allocation scheme offers 0.5, 1.2 and 0.6 dB gains over ZF, SIC and OSIC receivers, respectively. Both SIC and OSIC with power allocation outperform precoding schemes, e.g., at a BER of 10^{-3}, OSIC with power allocation offers 0.9 and 1.9 dB SNR gains over ZF-MBER and MMSE precoding/decoding, respectively. In Fig. 1, it is also observed that QPSK beamforming offers superior performance to all other simulated schemes except ML-MD precoding, e.g., at a BER of 10^{-3}, its SNR gain over OSIC with power allocation is 3.3 dB.

Fig. 2 illustrates average BER's in correlated Ricean fading channels. Performance of SIC without power allocation

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

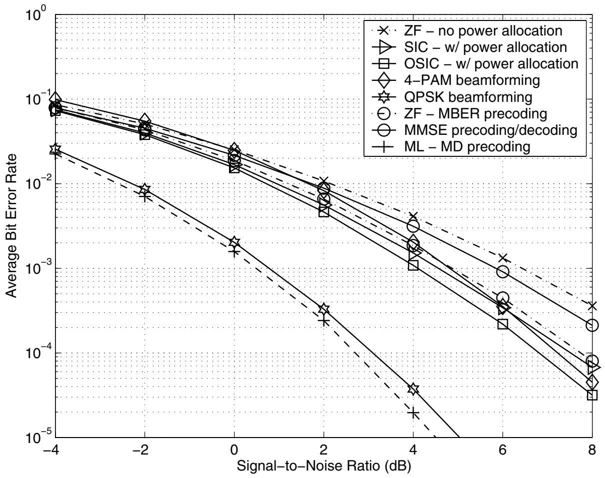

Fig. 1. Average BER in uncorrelated Rayleigh fading channel.

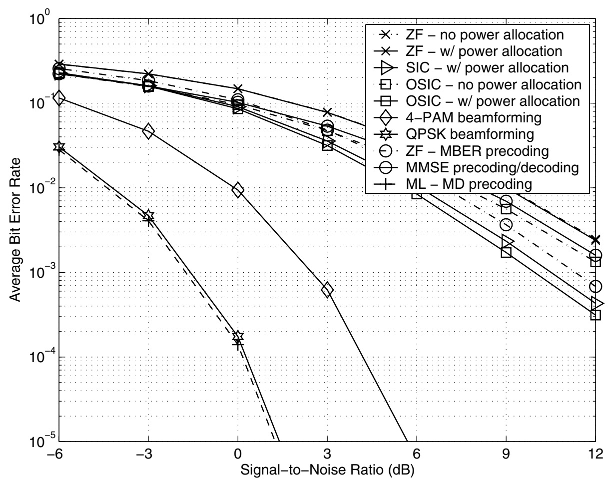

Fig. 2. Average BER in correlated Ricean fading channel ($K = 8$ dB).

(not shown) is nearly identical to that of MMSE precoding/decoding. Again, SIC and OSIC with power allocation outperform ZF-MBER and MMSE precoding/decoding. We also observe that the proposed 4-PAM and QPSK beamforming offer significant gain over the power allocation schemes shown: at a BER of 10^{-3}, 7.5 and 11.5 dB SNR gain over OSIC with power allocation are observed. This is as excepted since in Ricean fading, due to the existence of a line-of-sight (LOS) component, the channel matrix is likely to be ill-conditioned.

General Observations: From Figs. 1-2, we observe the SNR losses of QPSK beamforming relative to ML-MD precoding are less than 0.5 dB, and decrease as channel correlation increases.

VI. Conclusions

Minimum BER (MBER) transmit power allocation and beamforming for TIMO spatial multiplexing are proposed in this paper. It is shown that SIC and OSIC with approximate MBER (AMBER) power allocation outperform linear ZF-MBER and MMSE precoding/decoding schemes in Rayleigh fading channels, e.g., at a BER of 10^{-3}, OSIC with power allocation offers 0.9 dB and 1.9 dB SNR gains over ZF-MBER and MMSE precoding/decoding respectively. The proposed 4-PAM and QPSK transmit beamforming eliminates error floors and offers superior performance over both power allocation and ZF-MBER and MMSE precoding/decoding in Ricean fading channels, e.g., at a BER of 10^{-3}, their SNR gains over OSIC with power allocation are, respectively, 7.5 and 11.5 dB. Compared to more general precoding methods, the proposed transmit optimization schemes provide a simple and efficient way to exploit partial channel state information at the transmitter.

Acknowledgement

This research was made possible through support from Bell Mobility and Samsung.

References

[1] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bölcskey, "An Overview of MIMO Communications—A Key to Gigabit Wireless," *Proc. of IEEE,* vol. 92, no. 2, pp. 198-218, Feb. 2004.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity–Part I: System Description," *IEEE Trans. Commun.,* vol. 51, no. 11, pp. 1927-1938, Nov. 2003.

[3] D. A. Gore, R. W. Heath, Jr., and A. J. Paulraj, "Transmit Selection in Spatial Multiplexing Systems," *IEEE Commun. Lett.,* vol. 6, no. 11, pp. 491-493, Nov. 2002.

[4] G. J. Foschini, G. D. Golden, R. A. Valenzuela, P. W. Wolniansky, "Simplified Processing for High Spectral Efficiency Wireless Communication Employing Multi-Element Arrays," *IEEE J. Select. Areas Commun.,* vol. 17, pp. 1841-1852, Nov. 1999.

[5] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis and H. Sampath, "Optimal Designs for Space-Time Linear Precoders and Equalizers," *IEEE Trans. Signal Processing,* vol. 50, pp. 1051-164, May 2002.

[6] Y. Ding, T. N. Davidson, Z.-Q. Luo, and K. M. Wong, "Minimum BER Block Precoders for Zero-Forcing Equalization," *IEEE Trans. Signal Processing,* vol. 51, no. 9, pp. 2410-2423, Sept. 2003.

[7] D. J. Love and R. W. Heath Jr., "Limited Feedback Precoding for Spatial Multiplexing Systems," *Proc. IEEE Globecom 2003,* vol.4, pp. 1857-1861, San Francisco, CA, Dec. 2003.

[8] N. Wang and S. D. Blostein, "Minimum BER Power Allocation for MIMO Spatial Multiplexing Systems," *Proc. IEEE ICC 2005*, Seoul, Korea, May 2005.

[9] J. Akhtar and D. Gesbert, "A Closed-Form Precoder for Spatial Multiplexing over Correlated MIMO Channels," *Proc. IEEE Globecom 2003,* vol. 4, pp. 1847-1851, San Francisco, CA, Dec. 2003.

[10] A. Abdi and M. Kaveh, "A Space-Time Correlation Model for Multielement Antenna Systems in Mobile Fading Channels," *IEEE J. Select. Areas Commun.,* vol. 20, no. 3, pp. 550-560, Apr. 2002.

[11] D. Gesbert, H. Bölcskey, D. Gore, and A. J. Paulraj, "Outdoor MIMO Wireless Channels: Models and Performance Prediction," *IEEE Trans. Commun.,* vol. 50, no. 12, pp. 1926-1934, Dec. 2002.

[12] N. Wang and S. D. Blostein, "Power Loading for CP-OFDM over Frequency-Selective Fading Channels," *Proc. IEEE Globecom 2003,* vol. 4, pp. 2305-2309, San Francisco, CA, Dec. 2003.

[13] S. Zhou and G. B. Giannakis, "Adaptive Modulation for Multi-Antenna Transmissions with Channel Mean Feedback," *Proc. of IEEE ICC*, Anchorage, Alaska, vol. 4, pp. 2281-2285, May 11-15, 2003.

[14] N. Wang and S. Blostein, "Minimum BER transmit power allocation and beamforming for two-input multiple-output spatial multiplexing," submitted to *IEEE Trans. Veh. Technol.*, Dec. 2004.

[15] L. Collin, O. Berder, P. Rostaing, and G. Burel, "Optimal minimum distance-based precoder for MIMO spatial multiplexing," *IEEE Trans. Signal Processing,* vol. 52, pp. 617-627, Mar. 2004.

