Power Loading for CP-OFDM over Frequency-Selective Fading Channels

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***Abstract*— In this paper we compare cyclic prefix (CP) based single and multicarrier block transmission schemes in frequency-selective fading channels. Analytical comparison shows that at moderate-to-high signal-to-noise ratio (SNR), the uncoded error rate performance of multicarrier transmission is inferior to that of single carrier. We propose new minimum bit error rate (MBER) power loading algorithms for multicarrier transmission. It is also shown that a simpler approximate MBER (AMBER) power loading method has performance close to that of the optimum MBER scheme. Performance of a variety of methods is compared analytically and verified by simulations.**

I. Introduction

Broadband wireless communication signals suffer from serious intersymbol interference (ISI) due to time-dispersive frequency-selective channels. Multicarrier modulation (MC), especially Orthogonal Frequency Division Multiplexing (OFDM) using a cyclic prefix (CP) has been receiving growing interest in recent years as countermeasures of frequency selectivity of wireless channels due to simplified equalization in the frequency domain [1]. The time-domain equalization of conventional single carrier (SC) transmission usually has much higher complexity (especially when the channel delay spread is large). Recently, it has been shown that block SC transmission with similar block structure and frequency-domain equalization to CP-OFDM, has similar equalization complexity and performance as OFDM [2].

It has been shown that OFDM without error-control coding suffers from serious error floors in channels with nulls [3]. In this paper, we present a framework for comparison between CP-based MC and SC block transmission schemes. From our analysis, uncoded OFDM is inherently inferior to CP-SC block transmission with the same processing complexity. To overcome this problem, joint power and bit loading has been widely studied in wire-line applications, where the channel can be regarded as static [4]. Wireless systems, however, are time-varying. For simplicity of implementation, power loading alone may be employed as an alternative to joint power and bit loading. A minimum *aggregate* bit error rate (BER) power loading algorithm was proposed in [5], which minimizes the block error rate.

In this paper, motivated by MBER linear equalization [6], we propose a minimum *average* BER power loading algorithm. We also propose a simpler approximate MBER algorithm with closed-form optimization solution and performance close to the optimum MBER scheme. Power loading using other criteria, such as minimum mean squared error (MMSE) and equal gain (EG), are also considered. Analytical performance comparison among SC and a variety of MC schemes is presented. Simulations are provided to verify our analysis.

II. CP-OFDM and CP-SC

A serial stream of data, $s(n)$, is serial-to-parallel (S/P) converted into data blocks of size N, $\mathbf{s}(i) \stackrel{\text{def}}{=} [s(iN), s(iN+1), \cdots, s(iN+N-1)]^T$. For multicarrier transmission, the multicarrier modulation is performed via inverse fast Fourier transform (IFFT) to obtain the time-domain signal block, $\mathbf{F}^{\mathcal{H}}\mathbf{s}(i)$, where $\mathbf{F}$ is the $N \times N$ FFT matrix with the (n,k)-th entry $[\mathbf{F}]_{n,k} = \frac{1}{\sqrt{N}} e^{-j2\pi nk/N}$, and $(\cdot)^{\mathcal{H}}$ denotes matrix Hermitian. For single carrier system, no multicarrier modulation is performed. A guard interval in terms of CP of length G is then inserted between each block, and the resulting redundant cyclic-prefixed block is parallel-to-serial (P/S) converted and sent sequentially through the channel. The multipath propagation channel can be modelled as a finite impulse response (FIR) filter with tap vector $[h_0, h_1, \cdots, h_L]^T$ and additive white Gaussian noise (AWGN) $\eta(n) \sim \mathcal{N}(0, N_0)$. At the receiver, the first G entries, corresponding to the CP are removed. For simplicity of analysis, we assume white input and noise, and the length of the CP, G, is no less than the channel model order, L.

A. CP-OFDM

Without loss of generality, we consider one data block. The received block of CP-OFDM can be written as (after CP removal),

$$\mathbf{r}_{mc} = \mathbf{H}\mathbf{F}^{\mathcal{H}}\mathbf{s} + \boldsymbol{\eta}, \tag{1}$$

where $\mathbf{H}$ is an $N \times N$ circulant channel matrix with first column $\mathbf{h} \stackrel{\text{def}}{=} [h_0, \cdots, h_L, 0, \cdots, 0]^T$; and $\boldsymbol{\eta}$ is the $N \times 1$ AWGN vector. By assumption, $\mathbf{R}_{ss} \stackrel{\text{def}}{=} \mathrm{E}\{\mathbf{s}\mathbf{s}^{\mathcal{H}}\} = E_s\mathbf{I}$ and $\mathbf{R}_{\eta\eta} \stackrel{\text{def}}{=} \mathrm{E}\{\boldsymbol{\eta}\boldsymbol{\eta}^{\mathcal{H}}\} = N_0\mathbf{I}$. Performing multicarrier demodulation via FFT, we obtain the frequency-domain received block

$$\begin{aligned} \mathbf{r}_{mc}^f &\stackrel{\text{def}}{=} \mathbf{F}\mathbf{r}_{mc} \\ &= \mathbf{F}\mathbf{H}\mathbf{F}^{\mathcal{H}}\mathbf{s} + \mathbf{F}\boldsymbol{\eta} \\ &= \mathbf{D}\mathbf{s} + \boldsymbol{\eta}^f, \end{aligned} \tag{2}$$

where the superscript ^{f} denotes frequency-domain variable, and the last equality follows from the well-known property of circulant matrices, $\mathbf{F}\mathbf{H}\mathbf{F}^{\mathcal{H}} = \mathrm{diag}(\mathbf{F}\mathbf{h}) \stackrel{\text{def}}{=} \mathbf{D}$. The frequency-domain zero-forcing (ZF) and MMSE equalizers are given by, respectively [1],

$$\mathbf{G}_{zf}^{mc} = \mathbf{D}^{\dagger}, \quad \mathbf{G}_{mmse}^{mc} = \mathbf{D}^{\mathcal{H}} \left(\tfrac{1}{\gamma_s}\, \mathbf{I} + \mathbf{D}\mathbf{D}^{\mathcal{H}} \right)^{-1}, \qquad (3)$$

where $(\cdot)^{\dagger}$ denotes Moore-Penrose pseudoinverse, and $\gamma_s \stackrel{\text{def}}{=} E_s/N_0$. Note that both frequency-domain equalizers are diagonal matrices. So matrix inversion is trivial (only complex division, also known as one-tap equalization), which reduces the implementation complexity significantly.

B. Single Carrier with CP (CP-SC)

After CP removal in the CP-SC receiver, the received block can be written as

$$\mathbf{r}_{sc} = \mathbf{H}\mathbf{s} + \boldsymbol{\eta}, \qquad (4)$$

The ZF and MMSE time-domain equalizers are given by circulant matrices, respectively [1],

$$\mathbf{G}_{zf}^{sc} = \mathbf{H}^{\dagger}, \quad \mathbf{G}_{mmse}^{sc} = \mathbf{H}^{\mathcal{H}} \left(\tfrac{1}{\gamma_s}\, \mathbf{I} + \mathbf{H}\mathbf{H}^{\mathcal{H}} \right)^{-1}, \qquad (5)$$

for which there exist fast implementations based on FFT, also known as frequency-domain equalization [2].

C. Average Instantaneous BER Performance

For simplicity of analysis, we consider BPSK modulation.

In CP-OFDM, the one-tap equalizer outputs are given by, respectively, (for $k = 0, 1, \cdots, N-1$)

$$\left[\hat{\mathbf{s}}_{zf}^{mc}\right]_k = [\mathbf{s}]_k + \frac{1}{H_k}[\boldsymbol{\eta}^f]_k, \qquad (6a)$$

$$[\hat{\mathbf{s}}_{mmse}^{mc}]_k = \frac{|H_k|^2}{1/\gamma_s + |H_k|^2}[\mathbf{s}]_k + \frac{H_k^*}{1/\gamma_s + |H_k|^2}[\boldsymbol{\eta}^f]_k, \qquad (6b)$$

where $H_k \stackrel{\text{def}}{=} [\mathbf{F}\mathbf{h}]_k = \sum_{l=0}^{L} h_l e^{-j\frac{2\pi kl}{N}}$ is the channel frequency response of the k-th subcarrier. We see that the MMSE equalization output (6b) is a biased estimate of the transmitted signal, which, from (6a), is actually a scaled version of the ZF estimate. The decision-point SNR's are given by, respectively,

$$\gamma_{zf,k}^{mc} = \gamma_s |H_k|^2, \quad \gamma_{mmse,k}^{mc} = \gamma_s |H_k|^2 + 1. \qquad (7)$$

The above SNR's of MMSE equalization do not take into account the bias in the decision variable, and the increased biased SNR is an artifact of the SNR definition [7]. Since there is no inter-carrier interference (ICI), it can be shown that the error rate performance can be approximately determined by the unbiased SNR[1]. By scaling (6b) to remove the bias, we calculate the unbiased SNR for MMSE equalization

$$\gamma_{mmse-u,k}^{mc} = \gamma_s |H_k|^2, \qquad (8)$$

[1]The noise term in the decision variable is generally not AWGN. Nevertheless, assuming AWGN gives a good approximation.

which is exactly the same as that of ZF equalization. Therefore, from here on, only ZF equalization is considered in CP-OFDM. The average instantaneous BER is given by

$$P^{mc}(\mathbf{h}) = \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{2\gamma_s |H_k|^2}\right), \qquad (9)$$

where $Q(x) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2} dy$.

For single carrier modulation with CP, the ZF equalization output is

$$\hat{\mathbf{s}}_{zf}^{sc} = \mathbf{s} + \mathbf{H}^{\dagger}\boldsymbol{\eta}. \qquad (10)$$

Clearly, (10) is an unbiased estimate, and there is no ISI. The BER performance is determined by the decision-point SNR,

$$\begin{aligned} \gamma_{zf,k}^{sc} &= \frac{E_s}{N_0[(\mathbf{H}^{\mathcal{H}}\mathbf{H})^{\dagger}]_{k,k}} \\ &= \frac{\gamma_s}{\frac{1}{N}\sum_{l=0}^{N-1} \frac{1}{|H_l|^2}}, \quad \forall k. \end{aligned} \qquad (11)$$

Using (11), the average instantaneous BER is given by

$$P_{zf}^{sc}(\mathbf{h}) = Q\left(\sqrt{\frac{2\gamma_s}{\frac{1}{N}\sum_{l=0}^{N-1}\frac{1}{|H_l|^2}}}\right). \qquad (12)$$

For MMSE-equalized single carrier modulation with CP, the decision variable is

$$\hat{\mathbf{s}}_{mmse}^{sc} = \mathbf{B}\mathbf{s} + \mathbf{G}_{mmse}^{sc}\boldsymbol{\eta}, \qquad (13)$$

where $\mathbf{B} \stackrel{\text{def}}{=} \mathbf{G}_{mmse}^{sc}\mathbf{H} = \mathbf{H}^{\mathcal{H}}\left(\frac{1}{\gamma_s}\,\mathbf{I} + \mathbf{H}\mathbf{H}^{\mathcal{H}}\right)^{-1}\mathbf{H}$ is generally not diagonal, and (13) experiences ISI. In this case, the decision-point SNR cannot be used to calculate BER because the ISI term cannot be approximated as Gaussian. Since both $\mathbf{G}_{mmse}^{sc}$ and $\mathbf{H}$ are circulant, $\mathbf{B}$ is also circulant. The coefficients of the desired and interfering terms are the same for all symbols in the block, which can be calculated efficiently using an FFT. Denoting, respectively,

$$\underline{\Theta} \stackrel{\text{def}}{=} [\Theta_0, \cdots, \Theta_{N-1}]^T, \quad \Theta_k \stackrel{\text{def}}{=} \frac{|H_k|^2}{1/\gamma_s + |H_k|^2},$$

$$\underline{\theta} \stackrel{\text{def}}{=} [\theta_0, \cdots, \theta_{N-1}]^T \stackrel{\text{def}}{=} \mathbf{F}^{\mathcal{H}}\underline{\Theta},$$

we have

$$\begin{aligned} \mathbf{B} &= \mathbf{F}^{\mathcal{H}} \underbrace{\mathbf{D}^{\mathcal{H}}\left(\tfrac{1}{\gamma_s}\,\mathbf{I} + \mathbf{D}\mathbf{D}^{\mathcal{H}}\right)^{-1}\mathbf{D}}_{\mathrm{diag}\{\underline{\Theta}\}}\mathbf{F} \\ &= \mathrm{circulant}\{\underline{\theta}\}, \end{aligned} \qquad (14)$$

where circulant$(\underline{x})$ denotes a circulant matrix with first column $\underline{x}$. Therefore, θ_0 is the coefficient of the desired signal, and $\{\theta_k\}_{k=1}^{N-1}$ represent ISI coefficients. The noise variance in (13) is given by

$$\begin{aligned} \sigma_k^2 &= N_0[(\mathbf{G}_{mmse}^{sc})(\mathbf{G}_{mmse}^{sc})^{\mathcal{H}}]_{k,k} \\ &= \frac{N_0}{N}\sum_{l=0}^{N-1}\frac{|H_l|^2}{(1/\gamma_s + |H_l|^2)^2} \\ &\stackrel{\text{def}}{=} \sigma^2, \end{aligned} \qquad (15)$$

which is independent of k. Using Beaulieu series [8], we calculate

$$P_{mmse}^{sc}(\mathbf{h}) \approx \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m \text{ odd}}}^{M} \frac{1}{m} e^{-\frac{m^2\omega^2}{2}} \times \sin\left(\frac{m\omega\theta_0}{\sigma}\right) \prod_{k=1}^{N-1} \cos\left(\frac{m\omega\theta_k}{\sigma}\right), \quad (16)$$

where the choice of parameters M and ω is described in [8].

III. Power Loading in CP-OFDM

Power loading algorithms allocate the transmitted signal power across subcarriers under the constraint of constant power per block. Denote p_k^2 as the transmitted power of the k-th subcarrier ($k = 0, 1, \cdots, N-1$), and define the power loading matrix $\mathbf{P} = \text{diag}([p_0, p_1, \cdots, p_{N-1}]^T)$. The block power constraint can be normalized as

$$\text{trace}\{\mathbf{P}^2\} = \sum_{k=0}^{N-1} p_k^2 = N. \quad (17)$$

From (2), the received frequency-domain block is given by

$$\mathbf{r}_{pmc}^f = \mathbf{DPs} + \boldsymbol{\eta}^f. \quad (18)$$

With one-tap equalization, we obtain,

$$\hat{\mathbf{s}}_{pmc} = \mathbf{s} + (\mathbf{DP})^\dagger \boldsymbol{\eta}^f. \quad (19)$$

Using (8) and (9), the decision-point SNR of the k-th subcarrier is given by $\gamma_k^{pmc} = \gamma_s |H_k|^2 p_k^2$, and the average instantaneous BER

$$P(\mathbf{h}, \mathbf{P}) = \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{2\gamma_s |H_k|^2 p_k^2}\right). \quad (20)$$

Note that CP-OFDM without power loading is just a special case of the power-loaded CP-OFDM with $\mathbf{P} = \mathbf{I}$.

A. MBER Power Loading

By differentiating (20), we note that $\nabla_{p_k^2}^2 P > 0$. The Hessian matrix is diagonal with positive diagonal entries, and, therefore, positive definite. Hence, $P(\mathbf{h}, \mathbf{P})$ is a convex function in $\{p_k^2\}_{k=0}^{N-1}$. Also, it is readily verified that the block power constraint (17) defines a convex feasible region. Therefore, power-constrained MBER power loading

$$\begin{cases} \min P(\mathbf{h}, \mathbf{P}) = \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{2\gamma_s |H_k|^2 p_k^2}\right) \\ \text{subject to} \quad \sum_{k=0}^{N-1} p_k^2 = N \end{cases}, \quad (21)$$

is a convex programming problem, for which there exists a unique global minimum [9]. The solution $\{p_k^2\}_{k=0}^{N-1}$ is unique and satisfies [10] (for $k = 0, 1, \cdots, N-1$)

$$\sqrt{\frac{\gamma_s |H_k|^2}{p_k^2}} e^{-\gamma_s |H_k|^2 p_k^2} = \mu, \quad (22)$$

which can be solved numerically. The parameter μ is chosen numerically according to the total power constraint (17). This algorithm requires an iterative procedure to obtain the optimum solution and N nonlinear equations must be solved numerically in each iteration. Therefore it suffers from problems such as slow convergence and high computational complexity, which motivates our proposed AMBER algorithm in the next.

B. Approximate MBER (AMBER) Power Loading

Instead of solving (21) directly, we approximate the objective function to find a suboptimal solution. An expression for the approximate BER is given by [11]

$$P_b \approx \frac{1}{5} e^{-g\gamma_s}, \quad (23)$$

where g is a constellation-specific constant, e.g., for BPSK, $g_{\text{BPSK}} = 1$. Accordingly, we formulate the AMBER power loading problem as

$$\begin{cases} \min \frac{1}{5N} \sum_{k=0}^{N-1} \exp(-\gamma_s |H_k|^2 p_k^2) \\ \text{subject to} \quad \sum_{k=0}^{N-1} p_k^2 = N \end{cases}, \quad (24)$$

which is also a convex programming problem. Using Lagrange multipliers, we obtain the closed-form solution,

$$p_k^2 = \frac{\ln |H_k|^2 + \nu}{\gamma_s |H_k|^2}, \quad \nu \stackrel{\text{def}}{=} \frac{\gamma_s - \frac{1}{N}\sum_{l=0}^{N-1} \frac{\ln |H_l|^2}{|H_l|^2}}{\frac{1}{N}\sum_{l=0}^{N-1} \frac{1}{|H_l|^2}}. \quad (25)$$

However, the above solution does *not* take into account the inherent inequality constraints, $p_k^2 \geq 0, \forall k$, which are satisfied only if $\min_k \ \ln |H_k|^2 \geq -\nu$. Otherwise, the solution should be modified as [10]

$$p_k^2 = \left(\frac{\ln |H_k|^2 + \nu}{\gamma_s |H_k|^2}\right)_+, \quad (26)$$

where $(x)_+ \stackrel{\text{def}}{=} \max\{0, x\}$, and ν is chosen to satisfy the block power constraint (17). Note that the block power $\sum_k p_k^2(\nu) = \sum_k \left(\frac{\ln |H_k|^2 + \nu}{\gamma_s |H_k|^2}\right)_+$ is a piecewise-linear increasing function of ν, with breakpoints at $\{-\ln |H_k|^2\}_{k=0}^{N-1}$. Therefore ν is unique and can be readily determined. Comparing with the MBER power loading algorithm, the AMBER algorithm does not require numerical solution of nonlinear equations, which reduce the computational complexity significantly.

C. Equal-Gain (EG) Power Loading

Since the $Q(\cdot)$ function decreases very rapidly as its argument increases, the BER is usually dominated by a few terms in (20) with smallest $|H_k|$'s. Power loading algorithms try to pre-equalize the transmitted signal so that the equivalent gains of subcarriers, $|H_k| p_k$, are distributed more evenly. One empirical solution is to allocate the transmitted power so that all the gains are equal[2], i.e.,

$$\begin{cases} |H_k|^2 p_k^2 = \text{constant}, \quad \forall k \\ \text{subject to} \quad \sum_k p_k^2 = N \end{cases}, \quad (27)$$

[2]This scheme was also addressed in [4] as a high-SNR approximation to power loading.

The solution is given by

$$p_{eg,k}^2 = \frac{1}{\frac{1}{N}|H_k|^2 \sum_{l=0}^{N-1} \frac{1}{|H_l|^2}}, \quad (k = 0, \cdots, N-1). \tag{28}$$

D. MMSE Power Loading

From (19), for power loading under the constraints (17), we obtain

$$\text{MSE} = N_0 \sum_{k=0}^{N-1} \frac{1}{|H_k|^2 p_k^2}. \tag{29}$$

Using Lagrange multipliers, the constrained MMSE solution can be found as

$$p_{mmse,k}^2 = \frac{1}{\frac{1}{N}|H_k| \sum_{l=0}^{N-1} \frac{1}{|H_l|}}, \quad (k = 0, \cdots, N-1). \tag{30}$$

IV. Comparison of CP-OFDM and CP-SC

We compare the average instantaneous BER of ZF-equalized CP-SC and a variety of CP-OFDM schemes with closed-form BER expressions. For MMSE-equalized CP-SC and MBER power-loaded CP-OFDM, we instead use simulations.

First, we compare ZF-equalized CP-SC and CP-OFDM without power loading. From (9) and (12), the average BER expressions can be unified as

$$P(\{\xi_k\}_{k=0}^{N-1}) = \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{\frac{2\gamma_s}{\xi_k}}\right), \tag{31}$$

where ξ_k's for OFDM and SC are given by, respectively,

$$\xi_k^{mc} = \frac{1}{|H_k|^2}, \quad \xi_k^{sc} = \frac{1}{N} \sum_{l=0}^{N-1} \frac{1}{|H_l|^2} \stackrel{\text{def}}{=} \xi^{sc}. \tag{32}$$

From the second derivatives of (31),

$$\frac{d^2 P}{d\xi_k^2} = \frac{1}{2N} \sqrt{\frac{\gamma_s}{\pi \xi_k^5}} \left(\frac{\gamma_s}{\xi_k} - \frac{3}{2}\right) e^{-\frac{\gamma_s}{\xi_k}},$$

we know that the Hessian matrix is positive definite, as long as the decision-point SNR,

$$\gamma_k \stackrel{\text{def}}{=} \frac{\gamma_s}{\xi_k} > \frac{3}{2} \ (1.76 \text{ dB}).$$

We note that in practical systems, γ_k's is usually (much) larger than 1.76 (dB) at moderate-to-high SNR ($\gamma_s \gg 1$), and, therefore, $P(\{\xi_k\}_{k=0}^{N-1})$ is a convex function of ξ_k's. From (32), we know

$$\xi^{sc} = \frac{1}{N} \sum_{k=0}^{N-1} \xi_k^{mc}. \tag{33}$$

By Jensen's inequality, we conclude that

$$P_{zf}^{sc}(\mathbf{h}) \leq P^{mc}(\mathbf{h}). \tag{34}$$

For CP-OFDM with equal-gain power loading, we calculate the decision-point SNR (see (28))

$$\gamma_{eg,k}^{pmc} = \frac{\gamma_s}{\frac{1}{N} \sum_{l=0}^{N-1} \frac{1}{|H_l|^2}} \equiv \gamma_{zf,k}^{sc}. \tag{35}$$

Therefore, CP-OFDM with equal-gain power loading has the same performance as ZF-equalized CP-SC.

Consider MMSE power-loaded CP-OFDM. From (20), (30) and (31), we have $\xi_{mmse,k}^{pmc} = \frac{1}{|H_k|} \cdot \frac{1}{N} \sum_{l=0}^{N-1} \frac{1}{|H_l|}$. When $\gamma_s \gg 1$, by Taylor series expansion of $P(\{\xi_{mmse,k}^{pmc}\})$ at ξ^{sc}, it can be shown that

$$P_{zf}^{sc}(\mathbf{h}) \leq P_{mmse}^{pmc}(\mathbf{h}). \tag{36}$$

Similarly, it can be shown that, for AMBER power-loaded CP-OFDM, at moderate-to-high SNR,

$$P_{amber}^{pmc}(\mathbf{h}) \leq P_{zf}^{sc}(\mathbf{h}). \tag{37}$$

The proof is omitted due to lack of space.

V. Numerical Results and Discussions

Example 1: We verify the minimum BER ability of the MBER power loading algorithm, as well as the approximating ability of the AMBER scheme. For convenience of illustration, we consider the block size $N = 2$ system over two-tap channel $h_0^2 = 0.8, h_1^2 = 0.2$, with $\gamma_s = 20$ (13 dB). Power loading solution can be normalized as $\mathbf{P} = \sqrt{2}\ \text{diag}\{[\cos\theta, \sin\theta]^T\}$, where θ is determined by power loading schemes. For CP-OFDM without power loading, $\theta = \pi/4$, corresponding to $\mathbf{P} = \mathbf{I}$. Fig. 1 illustrates a polar plot of $-\log_{10}(P(\mathbf{h}, \theta))$ versus θ. As we can see, the MBER solution points in the minimum BER direction, and the AMBER solution offers a very good approximation to MBER solution. Both of them offer better performance than CP-OFDM without power loading.

Example 2: We compare the average instantaneous BER performance of single carrier and a variety of multicarrier schemes. The channel is randomly generated, with frequency response shown in the upper part of Fig. 2. The power loading coefficients of EG, MMSE, AMBER and MBER schemes are also shown in the lower part of Fig. 2. We can see that the AMBER solution is very close to that of MBER. The average

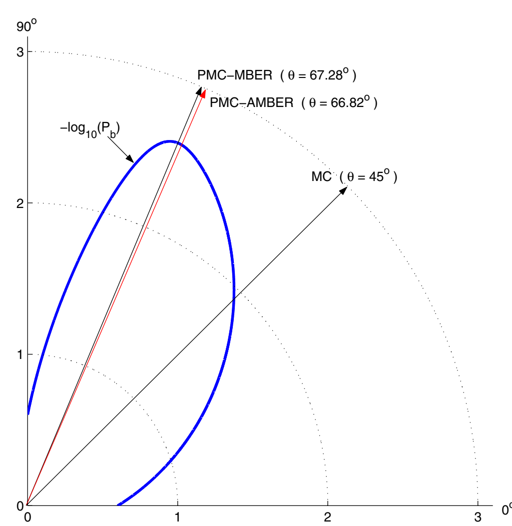

Fig. 1. A polar plot of $-\log_{10} P(\mathbf{h}, \theta)$ vs. θ.

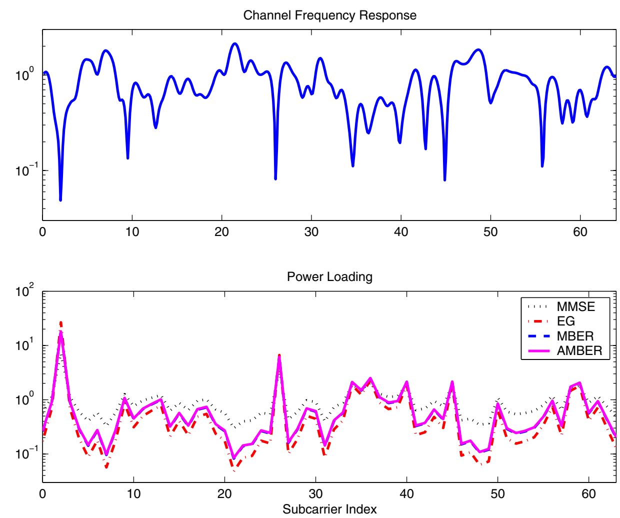

Fig. 2. Channel frequency response and corresponding power loading coefficients at $\gamma_s = 20$ (dB) ($N = 64$).

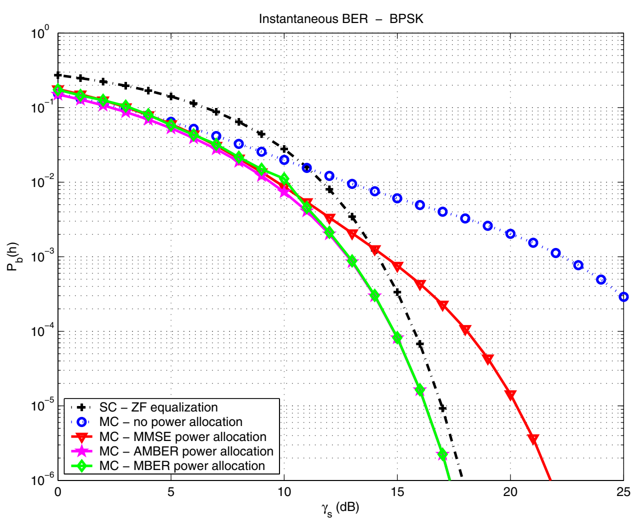

Fig. 3. Average instantaneous BER comparison of SC and MC ($N = 64$).

instantaneous BER performance of this channel is plotted in Fig. 3. At all SNR's shown, CP-OFDM with MBER power loading outperforms ZF-equalized SC (or OFDM with equal-gain power loading) and conventional CP-OFDM without power loading. Both MBER and AMBER algorithms have performance close to that of MMSE-equalized SC.

Remark 1: In MC systems, each symbol occupies a narrow spectral band and a long time duration, which makes MC more resistant to frequency-selective (multipath) fading. In SC systems, block equalization can be viewed as a form of diversity combining, which offers multipath diversity gain. For MC schemes, however, the equivalent channel is decoupled in the frequency domain. Multipath signals are non-resolvable in the frequency domain which means a loss of multipath diversity gain.

Remark 2: Recall that the power loading matrix $\mathbf{P}$ is constrained to be diagonal, in order not to introduce ICI. However, if we allow ICI in power loading, a better precoder can be found [12]. The processing complexity of such joint transmit and receive optimization is higher than the power loading algorithms. We note that the optimal precoder and decoder for MC and SC systems will converge since they solve the same problem in different domains.

VI. Conclusions

Cyclic-Prefix-based single carrier (SC) and OFDM multicarrier schemes are compared in this paper. It has been shown that uncoded CP-OFDM is inferior to CP-SC in frequency-selective channels. Power allocation algorithms are proposed for CP-OFDM. Analytical performance comparison and simulation results show that CP-OFDM with either MBER or AMBER power allocation offer superior performance to CP-SC with ZF equalization. As future work, performance optimization and comparison with imperfect channel knowledge will be considered.

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