Space-Time-Frequency Characterization of MIMO Wireless Channels

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Abstract—In this paper, we characterize the cross-correlation function (CCF) between the time-frequency transfer functions (TFs) of two sub-channels of a multiple-input multiple-output (MIMO) wireless fading channel. The TF of each sub-channel consists of a number of multi-path components caused by propagation of a transmitted signal in the random scattering media. The proposed CCF is expressed in terms of several environmental parameters (such as the moment generating function (MGF) of the delay profile (DP) and the pathloss exponent). It is a summation of two terms: the first term is due to the auto-correlation of multi-path components while the second term is due to the cross-correlation of multi-path components. Each term is a product of several correlation functions. Each of these correlation functions represents different dependencies of the wireless channel in terms of time, carrier frequency and the position of the antenna elements around both the transmitter and the receiver site. Interestingly, the last two terms of these functions are $1/2$-order (or $\eta$-order) integrations of the MGF of the DP, evaluated at two carrier frequencies (or the difference between carrier frequencies), where $\eta$ is the pathloss exponent of the environment.

Index Terms—Rayleigh fading, multipath fading channels, multielement antenna communication systems, multiple-input multiple-output (MIMO) systems, MIMO Rayleigh fading channels, antenna arrays, array configurations, electromagnetic wave scattering, radiowave propagation, Space Time and Frequency channel Characterization, Frequency division multiplexing, Multifrequency MIMO channel modeling, propagation path, Delay effects, Time-varying channels, Doppler effect, Doppler spread, isotropic scattering environment, omnidirectional antennas, correlation theory, space-time-frequency cross-correlation function, single-input single-output subchannels, fading correlation, Bessel functions of the first kind, Clark model, direction-of-arrival, direction-of-departure, land mobile radio, angular orientations, antenna separation, microcellular radio, time-delay, path-loss exponent, carrier frequency, channel gain, base station, mobile station, moment generating function, uniform phase change, $\eta$-order derivative, local scatterer, numerical calculations/simulations, probability density function.

I. INTRODUCTION

In order to design a multiple-input multiple-output (MIMO) wireless communication system, and to predict the impact of random multipath fading on its performance, it is necessary to have accurate and reliable MIMO channel models [1], [2]. Analyzing the statistical behavior of the channel transfer function (TF) of a wideband or narrowband communication system is a critical method to characterize MIMO systems [3]–[10]. In most environments, the response of the channel is composed of the superposition of responses of a large number of propagation paths. The central limit theorem suggests an asymptotically zero-mean Gaussian random process for the TF. Therefore, such a random process is often characterized by its second-order statistics, i.e., the cross-correlation function (CCF).

In order to derive the CCF of MIMO channels, motivating models based on the parameters of the propagation environment are discussed in the literature, e.g., [3]–[10]. Most of these models employ a certain geometry for the scatterers around the mobile station (MS), e.g., one-ring geometry. Alternatively in this paper, we establish a mathematical relation between the random time-delay and the random channel-gain associated with each scattered waveform. We use appropriate probability density functions (pdf) for the parameters such as the time-delay, the direction of arrival (DOA) and the direction of departure (DOD). We derive an expression for the space-time-frequency (STF) CCF between correlated links of a two-dimensional (2D) MIMO channel [11], [12]. This CCF characterizes the behavior of the MIMO channel in terms of the location of the antenna elements at both transmitter and receiver ends, the speed of the Mobile Station (MS), different time indices, different carrier frequencies, propagation delay profile (DP) and pathloss exponent. Specifically, the expression of the CCF is a summation of two terms: each term is a product of some Bessel functions and other terms. The other terms are $1/2$-order (or $\eta$-order) integrations of the moment generating function (MGF) of the DP evaluated at two carrier frequencies (or at the difference between two frequencies). The impact of the location of antenna elements appears in the operand of Bessel terms of the CCF. These zero-order Bessel functions are the direct result of the assumption of uniformly distributed directions (or energy) in the space [13]. The proposed model is efficient for analysis and simulation purposes and is a generalization of some existing isotropic models based on the Jake’s/Clarke model [12].

This paper is organized as follows: notations and assumptions are introduced in Section II; the proposed STF-CCF is derived in Section III; the behavior of the CCF is numerically evaluated in Section IV; concluding elements are brought together in Section V.

II. MIMO RAYLEIGH FADING CHANNEL

Figure 1 shows a pair of BS-MS antennas from a multi-element antenna communication system in a 2D propagation environment (for 3D propagation, see [15]). It also shows a moving MS with the constant speed $v(\frac{m}{sec})$ and a fixed BS in
the 2D plane. Throughout this paper, the superscripts $B$ and $M$ indicate variables at BS and MS sides respectively, and vectors are bold lowercase letters. Omnidirectional antenna elements are located around the MS and the BS local coordinates $O^B$ and $O^M$, with relative position vectors $\mathbf{a}_p^B$ and $\mathbf{a}_m^M$, respectively (for the impact of non-omnidirectional antennas in non-isotropic propagation, see [16]). We assume that the distance between scatterers and antenna arrays is much larger than the inter-element antenna distances such that propagation waveforms in the scattering environment are plane waves. We also assume that the number of propagation paths is large enough such that the channel is Rayleigh by virtue of the central limit theorem. Each antenna array element $\mathbf{a}_m^M$ receives the signal through the media via a number of dominant propagation paths with different lengths. Since, the line-of-sight (LOS) propagation path between the transmitter and the receiver can be treated separately [9], we assume no LOS.

By considering the received signal as a linear combination of plane waves [17], [18], we achieve a solution based on Maxwell’s equations. Each component of such a combination is the result of scattering in the propagation environment, i.e., each component is associated with a path gain $g_{p,m;i}$ and a path phase shift $\phi_i$. The path gain embodies pathloss shadowing and fast fading along the path. The path phase shift represents the phase change accumulated by antenna and scatterer interaction. The time-frequency TF of the subchannel consisting of the transmitter antenna element located at $\mathbf{a}_p^B$, the propagation environment and the receiver antenna element located at $\mathbf{m}_m^M$ is given by,

$$h_{pm}(t, \omega) = \sum_{i=1}^{I} g_{p,m;i} \exp(j\phi_i - j\omega \tau_{p,m;i}(t)), \quad (1)$$

where $I$ is the number of dominant paths resulting from scattering and $\tau_{p,m;i}(t)$ is the time-delay over $i^{th}$ path between $\mathbf{a}_p^B$ and $\mathbf{a}_m^M$. The propagation delay over $i^{th}$ path, $\tau_{p,m;i}(t) \triangleq \tau_{p,m;i} - \frac{c}{v^T \Theta_i^M}$, is time-varying due to the mobility of MS. Substituting the time-varying delay in (1), the channel TF is represented by,

$$h_{pm}(t, \omega) = \sum_{i=1}^{I} g_{p,m;i} \exp(j\phi_i + j\varpi_i t - j\omega \tau_{p,m;i}(t)), \quad (2)$$

where $\omega$ is the carrier frequency, $\varpi_i \triangleq \frac{c}{v^T \Theta_i^M}$ is the Doppler shift of the $i^{th}$ received waveform, and $v$ and $c$ are the MS velocity vector and the speed of light, respectively. The TF $h_{pm}(t, \omega)$ varies with time mainly due to the user mobility, i.e., the Doppler effect.

In this paper, we make the following assumptions:

A1) Probability distributions of azimuth DODs and DOAs are all uniform over $[0, 2\pi]$. This is possible by assuming an isotropic homogenous rich scattering environment with a planar wave propagation between the BS and the MS in the 2D space. A sufficient condition is that the plane wave assumption holds in small regions around the BS and the MS [17]. In addition, we assume that DOAs and DODs are independent from each other and from time-delays, $\tau_{p,m;i}$ [5]–[7] (for 2D non-isotropic propagation, see [16]).

A2) As a consequence of the planar wave propagation, the path phase shift $\phi_i$ accurately approximates $\phi_{p,m;i}$. We take into account the phase contribution of surrounding scatterers by a random phase change parameter, $\phi_i \sim U[-\pi, \pi]$. Since the path phase shifts, $\phi_i$, appear in (1) in the form of $\exp(j\phi_i)$, the correlations of $\exp\{j\phi_i\}$ over different paths, $E[\exp\{j(\phi_{i1} - \phi_{i2})\}]$, have impact on the channel behavior. In order to take into account the phase contribution of surrounding scatterers, we assume that these correlations are non-zero only for similar paths. Specifically, we assume that the value of $E[\exp\{j(\phi_{i1} - \phi_{i2})\}]$ is equal to $\kappa^2$ for similar paths and is zero for dissimilar paths, where the number of similar paths for a given path is limited by $I_{sim} < I$. For example, propagation waveforms in a hall-way results in a larger value for $\kappa_{sim}$ than propagation waveforms in a rural environment. The softness factor, $\kappa^2 \leq 1$, characterizes the effect of the environment on the phase correlation. For the sake of simplicity, we also assume that $\exp\{j(\phi_{i1} - \phi_{i2})\}$ is independent from the channel gain and the time-delay. See [19], [20] for more complicated and more realistic phase difference models.

A3) We decompose the $i^{th}$ path propagation delay, $\tau_{p,m;i}$, into three components: one major delay due to the distance between the BS and the MS, and two relative propagation delays with respect to local coordinates across the BS and the MS antenna arrays. This can be written in the following form,

$$\tau_{p,m;i} = \tau_i - (\tau_{p,i} + \tau_{m,i}), \quad (3a)$$

$$\tau_{p,i} \triangleq \mathbf{a}_p^T \mathbf{\Theta}_i^B / c, \quad (3b)$$

$$\tau_{m,i} \triangleq \mathbf{a}_m^T \mathbf{\Theta}_i^M / c, \quad (3c)$$

where $\tau_i$ represents delay between $O^B$ and $O^M$, and $\tau_{p,i}$ and $\tau_{m,i}$ represent relative propagation delays from antenna elements, $\mathbf{a}_p^B$ or $\mathbf{a}_m^M$, to corresponding coordinates, $O^B$ or $O^M$, respectively [8]. The time-delays $\tau_i$ are assumed to be independent and identically distributed (i.i.d.) random variables with Exponential distribution. Exponential distribution is a common distribution in outdoor environments with isotropic propagation [5], [19]. The pdf of the time-delay $\tau_i$ is $f_{\tau_i}(x) = \frac{1}{\kappa} e^{-\kappa x}$.
\[ \frac{1}{2} \exp \left\{ -\frac{\tau - \Theta}{\sigma} \right\}, \quad \forall x \geq \tau - \sigma, \quad \text{where} \quad \tau = E[t_i] \text{ is the mean value to specify the distance between the MS and BS (major propagation distance), and} \quad \sigma \text{ is the delay spread.} \]

The CCF is a function of sampling times \( R_{\tau, \omega} \).

\[ \rho_{p,m,i} \text{ and propagation delay, } \tau_{p,m,i} \text{ are random parameters and are functions of the length of path } i. \]

The relationship between \( g_{p,m,i} \) and the average path loss power \( P(\tau_{p,m,i}) \) is \( g_{p,m,i} \triangleq \sqrt{P(\tau_{p,m,i})} \) [8], [18], [21]. The term \( \frac{1}{\tau} \) is introduced to retain a constant power random process. The shadowing effect (or slow fading) is not considered in this paper [18, Page 181]. From experimental measurements, the relationship between the pathloss, \( \eta \) and the time-delay, \( \tau_{p,m,i} \) is characterized by [22]:

\[ P(\tau_{p,m,i}) \triangleq P_0 (\tau_{p,m,i})^{-\eta}, \]

where \( \eta \) is called the pathloss exponent, and \( P_0 \) is a constant. Depending on the propagation media, the pathloss exponent is measured between 2 and 6 [8], [22]. From (5), (3a) and the obvious fact that \( \tau_i \gg \max \{|\tau_p|,|\tau_m|\} \), we approximate \( P(\tau_{p,m,i}) \approx P(\tau_i) \) for all BS and MS antennas as:

\[ g_{p,m,i} \approx \eta_i = \frac{P_0}{\sqrt{T_1 \tau_i^{-2}}}. \]

### III. A New Space-Time-Frequency Cross-Correlation Function

Using above assumptions, we derive an expression for the CCF of the TFs \( h_{pm}(t, \omega) \) and \( h_{qm}(t, \omega) \) of two arbitrary subchannels of a MIMO channel:

\[ R_{pm,qm}(t_1, t_2; \omega_1, \omega_2) \triangleq E[h_{pm}(t_1, \omega_1) h_{qm}^*(t_2, \omega_2)]. \]

The CCF is a function of sampling times \( (t_1, t_2) \), carrier frequencies \( (\omega_1, \omega_2) \) and antenna elements \( (m,p,n,q) \). The expectation operation is performed over all independent random variables. This CCF provides essential information for the random process \( h_{pm}(t, \omega) \) at the (free) variables \( p,m,t \) and \( \omega \). For instance, in the presence of enough number of multipaths and by virtue of the central limit theorem, it is concluded that the TF is a Gaussian random process. Therefore, the above second-order statistics fully characterize statistical behavior of the channel. We refer to \( R_{pm,qm}(t_1, t_2; \omega_1, \omega_2) \) as the space-time-frequency (STF) CCF of the MIMO channel. Replacing (2) and (3) in (7), the STF-CCF is written as in (8) that is appeared at the top of next page, with:

\[ \Psi_{t_1} \triangleq \exp \left\{ j (\varphi_{t_1} t_1 + \omega_1 \left[ a^T_p \Theta^T_{\omega_1} + a^T_m \Theta^T_{\omega_2} \right]) \right\}, \]

\[ \Psi_{t_2} \triangleq \exp \left\{ j (\varphi_{t_2} t_2 + \omega_2 \left[ a^T_q \Theta^T_{\omega_1} + a^T_n \Theta^T_{\omega_2} \right]) \right\}, \]

where \( \varphi_{t_1} \triangleq \frac{\omega_1}{\omega_2} \varphi^T_{\omega_1} \) and \( \varphi_{t_2} \triangleq \frac{\omega_2}{\omega_1} \varphi^T_{\omega_2} \) are Doppler shifts with respect to \( \omega_1 \) and \( \omega_2 \). Relative locations of the antenna elements on MS or BS sides (i.e., \( a^T_m, a^T_n, a^T_p, a^T_q \) and \( a^T_B \)) \( \Psi_{t_1} \) and \( \Psi_{t_2} \) are functions of time, carrier frequency and the location (antenna indices); therefore, these Bessel components characterize some aspects of impacts of space, time and frequency on the CCF.

### Remark 2: The second term in the STF-CCF describes a...
\[ R_{\text{pm},g\text{n}}(t_1, t_2; \omega_1, \omega_2) = E \left[ \sum_{i_1, i_2 = 1}^I g_{p,m;i_1} g_{q,n;i_2} \exp \{ j(\omega_2 t_{i_2} - \omega_1 t_{i_1} + \phi_{i_1} - \phi_{i_2}) \} \Psi_{i_1} \Psi^*_{i_2} \right]. \] (8a)

\[ R_{\text{pm},g\text{n}}(t_1, t_2; \omega_1, \omega_2) = \frac{P_0}{I} \sum_{i_1, i_2 = 1}^I \left\{ E \left[ \exp \{ j(\phi_{i_1} - \phi_{i_2}) \} \right] E \left[ (\tau_{i_1} \tau_{i_2})^{\frac{\alpha}{2}} \exp \{ j(\omega_2 t_{i_2} - \omega_1 t_{i_1}) \} \right] E \left[ \Psi_{i_1} \Psi^*_{i_2} \right] \right\}. \] (9)

\[ R_{\text{pm},g\text{n}}(t_1, t_2; \omega_1, \omega_2) = P_0 J_0 \left( \frac{d_B}{c} \right) J_0 \left( \frac{d_{m,n}}{c} \right) \Phi^* \left( j(\omega_2 - \omega_1) \right) + P_0 J_{\text{sim}} \kappa^2 J_0 \left( \frac{d_B}{c} \right) J_0 \left( \frac{d_B^2}{c} \right) J_0 \left( \frac{d_{m,n}^2}{c} \right) \left( \Phi^* \left( -j\omega_1 \right) \Phi^* \left( j\omega_2 \right) \right), \] (10a)

\[ \Phi^* \left( \frac{\sigma}{2} \right) = \begin{cases} \left( \exp \left( \frac{\sigma}{2} \right) \right) E \left( \frac{\sigma}{2} - \alpha s \right), & \text{if } \eta = 2, \\ \left( \exp \left( \frac{\sigma}{2} \right) \right) E \left( \frac{\sigma}{2} - \alpha s \right) + \left( \exp \left( \frac{\sigma}{2} \right) \right) E \left( \frac{\sigma}{2} - \alpha s \right), & \text{if } \eta = 4, \\ \left( \exp \left( \frac{\sigma}{2} \right) \right) E \left( \frac{\sigma}{2} - \alpha s \right), & \text{if } \eta = 6, \end{cases} \] (11)

non-stationary behavior for the channel. The exceptional case is when \( \kappa^2 = 0 \) (see Assumption A2). In such a case the random process \( h_{\text{pm}}(t, \omega) \) is jointly stationary in time and space at the transmitter site, and space at the receiver site if \( \omega_1 = \omega_2 \). In this case, the CCF only depends on the time difference, \( t_1 - t_2 \) and the differences of the antenna position vectors, \( a_B^p - a_B^q \) and \( a_B^m - a_B^n \). If \( t_1 = t_2 \) and \( \omega_1 \neq \omega_2 \), and in the case of single antennas, the random process \( h_{\text{pm}}(t, \omega) \) is stationary in frequency \( \omega \).

**Remark 3:** The second term in (10a) describes the impact of the paths in which the phase shifts are dependent. The dependency is evaluated in terms of the softness factor \( \kappa^2 \), while the number of such paths is limited to \( I_{\text{sim}} < I \) [19], [20], [23]. The coefficient \( I_{\text{sim}} \kappa^2 \) plays an important role in the communication performance and is maximum at \( \kappa = 1 \), i.e., when the phase shifts are fully correlated.

**Remark 4:** Each summating term in (10a) consists of the product of a number of correlation functions. For instance, these functions for the second term are: \( J_0 \left( \frac{d_B}{c} \right) \), \( J_0 \left( \frac{d_{m,n}}{c} \right) \), \( J_0 \left( \frac{d_B^2}{c} \right) \), \( J_0 \left( \frac{d_{m,n}^2}{c} \right) \). In the above remark. This decomposition is the direct result of the independency of DODs and DOAs, introduces a Kronecker structure for the CCF (see [6], [24] and references therein). Such a decomposable form offers a better representation of the impact of different parameters on the behavior of the wireless channel. Evidently, dependent DODs and DOAs introduce more realistic and more complicated models.

**Remark 5:** The components \( \Phi^* \left( \frac{\sigma}{2} \right) \left( -j\omega_1 \right) \Phi^* \left( \frac{\sigma}{2} \right) \left( j\omega_2 \right) \) and \( \Phi^* \left( j(\omega_2 - \omega_1) \right) \) describe the impact of the DP and the pathloss exponent on the correlation of the channel TF. The CCF also depends on the carrier frequencies, \( \omega_1 \), \( \omega_2 \) via \( |d_B^p| \) and \( |d_B^q| \). Consistent with the literature [7], this fact shows that the CCF decreases when these frequencies or their differences increase.

**IV. NUMERICAL ILLUSTRATIONS**

In this section we numerically evaluate the magnitude of the CCF, \( |R| \), versus some key parameters. We also determine the Coherence Bandwidth (CB) in different environments. In the reported figures the unit for the antenna spacing is half the Coherence Bandwidth (CB) in different environments. In other graphs, we assume \( \tau = 3.33 \mu \text{s} \), the reported figures the unit for the mobile speed \( v = 20 \text{ km/h} \), pathloss exponent \( \eta = 2 \), softness factor \( \kappa^2 = 0 \), mean of the DP \( \tau = 3.33 \mu \text{s} \), \( f_1 = 1 \text{GHz} \), \( \Delta f = f_2 - f_1 \), \( a_B^m = 2[1 - 1]^2 \text{cm} \), \( a_B^p = [1 1]^2 \text{cm} \), \( a_B^q = 15[1 0]^2 \text{cm} \), \( a_B^r = 0 \text{cm} \).

![Fig. 2. Frequency selectivity for different values of the DP delay spread: CCF with respect to different delay spread values \( \sigma, t_1 = 0, t_2 = 1 \text{sec}, \) mobile speed \( v = 20 \text{ km/h}, \) pathloss exponent \( \eta = 2, \) softness factor \( \kappa^2 = 0 \), mean of the DP \( \tau = 3.33 \mu \text{s}, \) \( f_1 = 1 \text{GHz}, \) \( \Delta f = f_2 - f_1, \) \( a_B^m = 2[1 - 1]^2 \text{cm}, \) \( a_B^p = [1 1]^2 \text{cm}, \) \( a_B^q = 15[1 0]^2 \text{cm}, \) \( a_B^r = 0 \text{cm}. \) 

In Figure 2, we plot the CCF with respect to different delay spread values \( \sigma, t_1 = 0, t_2 = 1 \text{sec}, \) mobile speed \( v = 20 \text{ km/h}, \) pathloss exponent \( \eta = 2, \) softness factor \( \kappa^2 = 0 \), mean of the DP \( \tau = 3.33 \mu \text{s}, \) \( f_1 = 1 \text{GHz}, \) \( \Delta f = f_2 - f_1, \) \( a_B^m = 2[1 - 1]^2 \text{cm}, \) \( a_B^p = [1 1]^2 \text{cm}, \) \( a_B^q = 15[1 0]^2 \text{cm}, \) \( a_B^r = 0 \text{cm}. \)
Fig. 3. Spatial-Frequency selectivity: CCF with respect to the antenna spacing and the carrier frequency offset, $\Delta f = f_2 - f_1$, using Exponential DP with mean $\tau = 3.33\mu$sec, and delay spread $\sigma = 1\mu$sec, $t_1 = t_2 = 0$, pathloss exponent $\eta = 2$, softness factor $\kappa^2 = 0$, $a_{0}^{h} = [1 1] \lambda$, $a_{0}^{\beta} = [0 1] \lambda$, $a_{0}^{\delta} = 0cm$, $k \in [0, 4]$.

The increase in the delay spread $\sigma$ results not only from the Bessel functions (as the carrier frequencies appear in the Bessel operands), but also from the term produced by the MGF of the DP. For a single-input single-output channel, Figure 4 illustrates an almost linear relation between measurements in the literature [17], [18], [25], [26]. In a log-log scale, Figure 4 shows the CB with respect to the time-delay spread $\sigma$, considering two different definitions of the CB; using Exponential DP with mean $\tau = 3.33\mu$sec, $t_1 = t_2 = 0$, $f_1 = 1GHz$, $\Delta f = f_2 - f_1$, softness factor $\kappa^2 = 0$, $a_{0}^{h} = 0cm$, $a_{0}^{\beta} = 0cm$, $a_{0}^{\delta} = 0cm$, $\eta = 2$, $\eta = 4$, $\eta = 6$, for different propagation environments with different pathloss exponents $\eta$: $\eta = 2$ (typical urban), $\eta = 4$ (crowded urban) and $\eta = 6$ (rural).

### V. Conclusions

We propose a simple and closed-form mathematical expression for the cross-correlation function (CCF) of a completely diffuse multiple-input multiple-output (MIMO) wireless channel with 2D azimuth propagation environment. This closed-form function is the direct result of independency between transmitter and receiver scattering, i.e., independent direction of departures (DOD) and direction of arrivals (DOA) at both the transmitter and the receiver. This function is composed of two summating terms. The first term represents the auto-correlation between multi-path components, while the second term represents the cross-correlation between them. Each term is decomposed into a product consisting of a number of correlation functions. Interestingly, the $\eta$th-order integration (or the $\frac{\eta}{2}$th-order integration) of the moment generating function (MGF) of the delay profile (DP) evaluated at the difference of two carrier frequencies (or at two carrier frequencies) is one of the correlation functions, where $\eta$ is the environment pathloss exponent. In other words, the joint impact of the pathloss exponent, the carrier frequencies and the DP are explicitly characterized by this term. These carrier frequencies also appear in other terms of the CCF: in the operand of some other multiplicative Bessel functions. Moreover, these Bessel terms are functions of the spatial location of the antenna elements (either at the transmitter or at the receiver), time, carrier frequency and speed; therefore, this CCF is a generalization of the Jake’s/Clarke model for MIMO environments with uniformly distributed DOAs and DODs. Our proposed model accurately predicts the CB for some outdoor propagation environments by $\text{CB} = k_1 \sigma^{k_2}$.

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APPENDIX: CALCULATION OF THE CROSS-CORRELATION FUNCTION

In this appendix, we derive the following terms:

\[
E \left[ (\tau_i, \tau_j)^2 \exp\left( j (\omega_2 \tau_i - \omega_1 \tau_j) \right) \right], \\
E \exp(j (\phi_i - \phi_j)), \\
\Phi_i \Phi_j \right),
\]

that are involved in the summation (9). If \( i = i_2 \), some of the random variables in this expression become identical; therefore, we consider two cases \( i_1 \neq i_2 \) and \( i_1 = i_2 \). From assumptions A2, we have:

\[
E \exp(j (\phi_i - \phi_j)) =
\begin{cases}
\kappa^2, & i_1 \neq i_2 \text{ & for two similar paths}, \\
0, & i_1 \neq i_2 \text{ & for two dissimilar paths}, \\
1, & i_1 = i_2,
\end{cases}
\]

\[
E \left[ (\tau_i, \tau_j)^2 \exp\left( j (\omega_2 \tau_i - \omega_1 \tau_j) \right) \right] =
\begin{cases}
\Phi_i^{(n/2)}(j \omega_2) \Phi_i^{(n/2)}(-j \omega_1), & i_1 \neq i_2, \\
\Phi_j^{(n)}(j \omega_2 - \omega_1), & i_1 = i_2,
\end{cases}
\]

where the MGF of the phase change is \( \Phi_i(s) = \exp(s) - \exp(-s) \). From (8b) and other assumptions, we get (15) which appeared at the top of next page.

REFERENCES


Coherence Bandwidth: $4$

Spatial-Frequency selectivity: $2$

Frequency selectivity for different values of $1$

A 2D-MIMO propagation environment:

\[ E \Psi_i \Psi_j = E \exp\left(\frac{j\omega_1}{c} ([a_i^M + t_1 v]^T \Theta_{11}^M + a_p^T \Theta_{11}^B) - \frac{j\omega_2}{c} ([a_i^M + t_2 v]^T \Theta_{12}^M + a_q^T \Theta_{12}^B)\right), \]

\[ = \left\{ \begin{array}{l}
E e^{\frac{j\omega_1}{c} [a_i^M + t_1 v]^T \Theta_{11}^M} E e^{\frac{j\omega_2}{c} [a_i^M + t_2 v]^T \Theta_{12}^M} E e^{\frac{j\omega_2}{c} [a_i^M + t_2 v]^T \Theta_{12}^B}, \\
J_0 \frac{|\omega_1 (a_i^M + t_1 v)|}{c} J_0 \frac{|\omega_2 (a_i^M + t_2 v)|}{c}, \quad i_1 \neq i_2,
\end{array} \right. \]

\[ J_0 \frac{|\omega_1 (a_i^M + t_1 v)|}{c} J_0 \frac{|\omega_2 (a_i^M + t_2 v)|}{c}, \quad i_1 = i_2. \]

(15)

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