A CFAR Detector in a Non-Homogenous Weibull Clutter

A. Pourmottaghi, M. R. Taban and S. Gazor

Abstract—We propose a method that improves the CFAR detection of targets in a non-homogenous clutter environment. We assume that the clutter data from neighboring cells may have an edge with unknown location dividing data samples into two parts with different iid Weibull distributions. We propose automatic clutter edge localization which allows us to eliminate the outlier/leading data, and thereby improves the CFAR detection performance. We also suggest a CFAR detector which uses this estimator.

Index Terms—CFAR, Weibull, clutter edge, non-homogenous clutter.

I. INTRODUCTION

constant False Alarm Rate Detectors (CFARDs) are frequently used in radar target detection. In these schemes, the received data is passed through an envelope/square law detector and is compared with an adaptive threshold. This adaptive threshold is calculated using data collected from reference cells, i.e., the cells surrounding the Cell Under Test (CUT) [1], [2]. Usually, the reference data are assumed to be independent and identically distributed (iid) [3], [4]. There are many CFAR algorithms proposed for radar detection and most of them are derived assuming a Gaussian distribution for clutter [5]–[10]. However in general, it is known that the clutter distribution is not Gaussian, in particular for high resolution radars and low grazing angles [11]–[13]. The non-Gaussian clutter distributions have often several unknown parameters which may need to be estimated for setting the threshold [14]. For such clutter distributions, the conventional CFARDs (such as Cell Averaging CFAR [15] and Ordered Statistic CFAR (OS-CFAR) [16]) cannot maintain their false alarm rate at a constant desirable value (see [17] and references therein). As a remedy, several methods are proposed in order to improve the detection performance for non-Gaussian clutter and maintain their false alarm rate [17]–[19]. The Weibull probability density function (pdf) is one of the non-Gaussian distributions commonly used for clutter modeling because it fits the empirical data very well [20]. In [18], [21], [22], some CFAR target detection methods are proposed for Weibull clutter assuming known shape parameter. Under this assumption, an optimal detector is proposed in [21]. In addition, by using this pdf, the manipulation and analysis are easy and tractable for CFAR detection.

Goldstein proposed the log-t CFARD for Log-normal and Weibull environments [17]. In homogenous clutter, this detector is optimal for the Log-normal clutter and performs well for the Weibull clutter. The Weber-Haykin CFARD (WH-CFARD) algorithm [19] is an extension of OS-CFARD for two parameters distributions such as Weibull and Log-normal clutters, which is further extended in [23]–[25]. Our simulation results show that the log-t detector often outperforms the WH-CFARD in homogenous clutter. In contrast, the WH-CFARD usually outperforms the log-t detector in the presence of interfering targets within the reference samples. For the Weibull clutter, Maximum Likelihood CFARD (ML-CFARD) is derived in [18] and considerably outperforms previous detectors in homogenous clutter. This detector is computationally expensive and its performance is more sensitive to the homogeneity of the environments [18]. In some recent CFARDs, e.g. in [25]–[28], the reference cells are divided into leading and lagging windows. These two windows are processed separately and their outputs are combined for setting the threshold.

Unfortunately, all of the detectors listed above perform well only in some assumed non-homogenous environments and their performance degrades significantly around cells where the texture of clutter changes, e.g., from water to land. In this paper, to improve the detection performance around those edges, we assume that the clutter data may have two parts with separate iid Weibull pdfs with unknown parameters and an unknown edge location.

Several extended CFARDs are proposed attempting to improve the detection performance and maintain the false alarm rate for specific non-homogenous clutter environments (e.g., see [3], [5] and references therein). Therefore, these methods are effective for the specific environments for which they are designed, however, they may be deficient in other cases. In particular, the performance of these methods is poor for the cells which are located around the clutter edges. In this paper, we only consider the non-homogeneity due to the edges around the border of two homogenous environments and propose a clutter edge detection algorithm for filtering outlier data. This edge detection allows us to reduce the impact of the portion of the clutter data from the neighboring environment on the performance of target detection. In [29], a clutter edge detection method is proposed for the radar CFAR detection assuming Gaussian clutter. Using a Bayesian approach, they select a homogenous region around the cell under the test. Then, they use this region to determine the detection threshold of a conventional CFARD. The proposed algorithm in [29] is restricted to Gaussian clutter and also requires to solve three nonlinear equations iteratively using Expectation-Maximization (EM) algorithm. Doyuran and Tanik have developed the mentioned EM CFARD for the Weibull clutter in [30] and proposed a CFAR detector in range-heterogeneous Weibull clutter in [31]. Their algorithm tests the homogeneity of clutter on the reference window using Anderson-Darling Goodness-of-fit test for Weibull distribution and estimates the position of sharp transition on the reference window. This algorithm involves a complex iterative procedure for estimating clutter parameters.

In this paper, we propose an algorithm for detection and localization of a possible edge within clutter reference data. We also propose an enhanced detector by incorporating the estimated edge location in a log-t detector. The proposed detector is CFAR and robust against clutter edge, i.e., its false alarm probability \( P_{fa} \) deviates significantly less around the clutter edge compared with that of other existing state-of-the-detectors. Moreover, the detection probability is improved using the proposed detector. The main idea is to estimate the location of a possible edge within data and calculate the threshold using only the region of clutter data which most likely has similar statistics as CUT’s. Our simulations reveal that the proposed detector performs similar to the log-t CFARD in homogeneous clutter. In addition, the proposed method significantly outperforms the log-t, WH and ML CFARDs in presence of a clutter edge, i.e., it provides an enhanced detection probability with smaller variations in \( P_{fa} \). The virtue of the proposed detector is the considerable improvement of the detection performance around clutter edge.
In Section II, we propose a clutter edge detection method by approximating the nonlinear ML equations. In Section III, we suggest a robust CFARD employing the output of this clutter edge detector in order to enhance the detection performance around clutter edges. In Section IV, the proposed detector is evaluated and compared with the log-t and WH CFARs. Finally, we conclude our discussion in Section V.

II. ESTIMATION OF CLUTTER EDGE POSITION

In this section, we propose an algorithm for detecting a possible edge and estimating its position within the reference window. This algorithm is derived as a pre-processor in order to improve the performance of a conventional CFARD. We assume that an output sample $x_n$ from an envelope/square law detector is available for the $n$th cell within a reference window of size $2N$. We assume that these random samples $x = [x_0, x_1, \ldots, x_{N-1}, x_{N+1}, \ldots, x_{2N}]^T \in \mathbb{R}^{2N}$ are independent. Note that, the data sample $x_{2N}$ (denoted hereafter by $x_{\text{CUT}}$) from the CUT is excluded from the reference data since the CUT is tested for a possible target. We assume that the window size is small enough compared with the rate of changes in the clutter statistics such that the probability of having more than one edge within one window is very small, i.e., either there is no edge or at most one edge with unknown location within the reference window. Figure 1 illustrates the arrangement of radar samples corresponding to this sliding window and a clutter edge at $M$. The logarithm is a one-to-one invertible function which transforms the Weibull clutter and as this sliding window and a clutter edge at $M$. We assume that elements of $y_{2N}$ have identical and independent distributions, i.e., $f(y_{2N}|a_{y,M}) = \prod_{k=1}^{2N} f(y_k|a_{y,M})$ where $y_k$ is the $k$th element of $y_{2N}$ and $a_{y,M}$ is the vector of unknown parameters. Similarly for $z_{2N}$, we use $f(z_{2N}|a_{z,M}) = \prod_{k=1}^{2N} f(z_k|a_{z,M})$ where $z_k$ is the $k$th element of $z_{2N}$. Therefore the joint probability density function of $x$ is given by $f(x|a_{y,M}) = \prod_{k=1}^{2N} f(y_k|a_{y,M}) \prod_{k=1}^{2N} f(z_k|a_{z,M})$. In other words, the vector parameters $a_{y,M}$ and $a_{z,M}$ can be different in these two partitions.

The ML estimate of unknown parameters $a_{y,M}$, $a_{z,M}$ and $M$ are obtained by maximizing $f(x|a_{y,M}, a_{z,M}, M)$, i.e.,

$$\hat{a}_{y,M}, \hat{a}_{z,M}, \hat{M} = \arg\max_M \left( \sup_{a_{y,M}} \sum_{k=1}^{M} \ln f(y_k|a_{y,M}) + \sup_{a_{z,M}} \sum_{k=1}^{2N-M} \ln f(z_k|a_{z,M}) \right).$$ (1)

For some distributions such as Gaussian, exact or approximate solution of the internal maximization in the above can be easily found and then maximized over $M$ [32]. In the following, we solve this problem for Weibull distribution, $f(x; B, C) = \frac{C x^{C-1}}{B^C} e^{-(x/B)^C}$ for $x > 0$, where $B$ and $C$ are the scale and shape parameters respectively.

The Weibull distribution is known to fit well to radar clutter data [20]. In this case, we rewrite the problem in (1) for logarithm of the input data. In fact, we assume that $\ln x_n$ denote the logarithm of clutter with Weibull distribution. It is well known that the logarithm of a random variable with a Weibull distribution has a Gumbel distribution, i.e., $f(x|a) = \frac{1}{a} \exp \left\{ \frac{x-b}{a} - \exp \left\{ \frac{x-b}{a} \right\} \right\}$, where $a = [a, b]$, $a = \frac{1}{\gamma}$ and $b = \ln B$ are the parameters of Gumbel distribution. In this case, we can write

$$\ln f(x|a) = -\ln a + \frac{x-b}{a} - e^{\frac{x-b}{a}}.$$ (2)

where $f(x|a)$ is the Log-Likelihood Function (LLF) of one of observed samples with Gumbel distribution and unknown parameters. Note that the LLC terms in (1) are in the above form. Then under $H_M$, we set the derivatives of (1) to zero with respect to unknown parameters to find ML estimates. By manipulating the results, it is easy to show that the ML estimate of unknown parameters is estimated using only one observed sample. We encounter the same problem for $M = 2N - 1$ for the second partition. Thus hereafter, we only consider that $2 \leq M \leq 2N - 2$.

To compute the MLE of unknown parameters the set of nonlinear equations in (3) and (4) must be solved iteratively for various values of $M$. Using $\hat{a}_{y,M}$ and $\hat{b}_{y,M}$ in (2) the maximum likelihood value in the first partition is given by

$$\ln f(y_i|\hat{a}_{y,M}, \hat{b}_{y,M}) = \sum_{k=1}^{M} \ln f(y_k|\hat{a}_{y,M}, \hat{b}_{y,M}),$$

$$= \sum_{k=1}^{M} \left( -\ln \hat{a}_{y,M} + \frac{y_k - \hat{b}_{y,M}}{\hat{a}_{y,M}} - e^{\frac{y_k - \hat{b}_{y,M}}{\hat{a}_{y,M}}} \right).$$

In order to derive a simple non-iterative solution, we substitute $\hat{y}_{M}$ from (5) into the last term of equation (5), i.e., in $e^{M \frac{y_k - \hat{b}_{y,M}}{\hat{a}_{y,M}}}$. Simplifying the expression, we obtain

$$\ln f(y_i|\hat{a}_{y,M}, \hat{b}_{y,M}) = -M (1 + \ln \hat{a}_{y,M}) + \sum_{k=1}^{M} \frac{y_k - \hat{b}_{y,M}}{\hat{a}_{y,M}}.$$ (6)

To approximate the above value, we use the following relationships between the parameters and the moments of Gumbel distribution [14], [33]

$$a = \frac{\sqrt{6}}{\pi} \sqrt{\text{var}(x)},$$

$$b = \text{E}\{x\} - \alpha,$$

where $\gamma \approx 0.57721$ is Euler-Mascheroni constant, $\text{E}\{\cdot\}$ and $\text{var}\{\cdot\}$ stand for the expectation and variance of a random variable, respectively. Motivated by (7) and (8), we propose the following estimators

$$\hat{\pi}_{y,M} = \frac{\sqrt{6}}{\pi} \sqrt{\text{std}(y_{2N})},$$

$$\hat{\pi}_{y,M} = \frac{\sqrt{6}}{\pi} \sqrt{\text{std}(y_{2N})},$$ (9)

$$\hat{\pi}_{y,M} = \frac{\sqrt{6}}{\pi} \sqrt{\text{std}(y_{2N})},$$ (10)

where $\pi_M = \frac{1}{M} \sum_{k=1}^{M} y_k$ and std$(y_{2N}) = \sqrt{\frac{1}{M-1} \sum_{k=1}^{M} (y_k - \pi_M)^2}$ are ensemble average and standard deviation of observed data. By substituting the above estimators into (6), we have

$$\ln f(y_i|\hat{a}_{y,M}, \hat{b}_{y,M}) = -M \left( 1 + \ln \left( \frac{\sqrt{6}}{\pi} \sqrt{\text{std}(y_{2N})} \right) \right) + \sum_{k=1}^{M} \frac{y_k - \pi_M}{\hat{a}_{y,M}}.$$ (11)
and since \( \sum_{k=1}^{M} \left( \frac{y_{k}}{\sigma_{y_{k},M}} \right) = \gamma M \), we obtain

\[
\ln f(y_{M} | \overline{y}_{y_{M}}, M) \approx M(\theta - \ln \{\text{std}(y_{M})\}),
\]

where \( \theta = \gamma - 1 - \ln(2\pi) \) is a constant number. Similarly, we can easily write the following approximation for the second partition

\[
\ln f(z_{M} | \overline{z}_{y_{M}}, M) \approx (2N - M)(\theta - \ln \{\text{std}(z_{M})\}).
\]

By substituting (12) and (13) in (1) and simplifying the result, we propose the following edge detector

\[
\tilde{M} \approx \arg \min_{2 \leq M \leq 2N-2} \left\{(2N - M) \ln(\text{std}(z_{M})) + M \ln(\text{std}(y_{M}))\right\},
\]

Note that, the above decision rule determines the most likely location of the edge provided that there is one. This decision is performed using the standard deviations and thus requires \( 2N(2N - 2) \) multiplications. However, this complexity can be significantly reduced using the fact that our window is sliding over range cells.

In [34], it is shown, by computer simulation, that the variance of estimators in (3) and (4) tends to the Cramér Rao lower bound for large number of samples. These estimators are not only asymptotically efficient but also consistent. Moreover their simulations also demonstrate that the bias of these estimators have a insignificant effect on the estimation error bounds. In addition, it is proven that the estimators in (9) and (10) are consistent [35] and their biases tend to vanish as the number of samples increases. Therefore, we conclude that (12) and (13) give consistent estimator for the log-likelihood functions with an insignificant bias.

Now we must test \( H_{\tilde{M}} \) against \( H_{0} \) to determine either there is an edge or not. To this end we compare the LLF of \( H_{\tilde{M}} \) with that of \( H_{0} \). However, simulation results reveal that \( H_{\tilde{M}} \) is always favored. This is intuitively justified as \( H_{\tilde{M}} \) represents a more complex model. Thus motivated by the Minimum Description Length (MDL) approach [36], we propose to add a penalty term in this comparison as follows,

\[
\ln f(\mathbf{x} | \overline{y}_{y_{M}}, \overline{z}_{z_{M}}, \tilde{M}) \approx \ln f(\mathbf{x} | \overline{y}_{y_{M}}, \overline{z}_{z_{M}}, \tilde{M}) - \ln f(\mathbf{x} | \overline{y}_{y_{M}}, \overline{z}_{z_{M}}, \tilde{M})\]

\[
(2N - \tilde{M}) \ln(\text{std}(z_{\tilde{M}})) + \tilde{M} \ln(\text{std}(y_{\tilde{M}})) - 2N \ln(\text{std}(\mathbf{x})) - \xi_{H_{\tilde{M}} - H_{0}},
\]

where \( \xi \) depends on the number of unknown parameters and is introduced to make the decision consistent. The value of \( \xi \) is often proportional with the logarithm of number of samples \( 2N [37] \). The value of \( \xi \) is also referred to as the difference of the code lengths required to describe the set of unknown parameters under \( H_{0} \) and under \( H_{\tilde{M}} \) and thus could be proportional with the difference between the number of unknown parameters in \( H_{0} \) and \( H_{\tilde{M}} \). In our problem, the number of unknown parameters is 4 for all \( H_{\tilde{M}} \) and is 2 for \( H_{0} \). Thus, we propose to use \( \xi = \lambda \ln(2N) \) and from computer simulation, we find that \( \lambda = 2 \) yields satisfactory results.

Note that the above criterion can not evaluate \( M = 1 \) or \( M = 2N - 1 \) since the standard deviation of a single sample is zero. Fortunately, these cases represent the exclusion of only one sample which does not have any significant impact on the target detection performance.

To evaluate the proposed estimator in various conditions, we compute the error \( e_{M} = \frac{1}{N} \sum_{k=1}^{N} \left| \frac{\tilde{M} - M}{M} \right| \) and the probability of missing to detect a clutter edge \( P_{\text{miss}}(M) \) for a given clutter edge location \( M \), where \( K \) is the number of the simulation iterations. Figure 2 plots the curves of \( e_{M} \) versus \( M \) for several values of the CCR. The clutter edge is located at \( \tilde{M} \) where

\[
\tilde{M} = \arg \min_{2 \leq M \leq 2N-2} \left\{(2N - M) \ln(\text{std}(z_{M})) + M \ln(\text{std}(y_{M}))\right\},
\]

Fig. 2. Error value of proposed edge estimator versus edge number \( M \) for \( C_{y}=0.7, C_{z}=1.6, B_{y}=1, 2N = 32 \) and five values of CCR=-10, -5, 0, 5 and 10 dB.

The computational routine of the proposed automatic edge detector is summarized as follows

- Start with reference data \( \mathbf{x} = [x_{1}, \ldots, x_{2N}]^{T} \) with length \( 2N, \xi = 2 \ln(2N) \).
- For \( m = 1, 2, \ldots, 2N - 2 \) compute
  \[
f(m) = \left\{ \begin{array}{ll}
  2N \ln(\text{std}(\mathbf{x})) - \xi, & \text{for } m = 1, \\
  m \ln(\text{std}(\mathbf{y})) + (2N - m) \ln(\text{std}(\mathbf{z})) & \text{for } 2 \leq m \leq 2N - 2,
  \end{array} \right.
  \]

- The clutter edge is located at \( \tilde{M} = \arg \min_{m} f(m) \). The clutter edge doesn’t exist if \( \tilde{M} = 1 \).
Error $e_M$ and miss probability $P_{\text{miss}}(M)$ of proposed edge estimator versus $\lambda$ for $[B_g, C_g] = [1, 0.7]$, $[B_s, C_s] = [10, 1.6]$ and various values of $2N = 24, 32, 48, 64, 72$ and $M = 1.5N$. a) $e_M$, b) $P_{\text{miss}}(M)$.

- Eliminate the outlier data: for $\hat{M} = 0, 1$, and $2N - 1$ use $w = x$, for $\hat{M} \geq N + 1$ use $w = y$ otherwise use $w = x$. Note that $w$ is used to calculate the detection threshold of the proposed detector.

III. CFAR TARGET DETECTION

The proposed detector is summarized as follows. We first apply the proposed clutter edge detector given by (14) and (15) as a pre-filter to select the proper subset of clutter data which shares the same clutter parameters as the CUT and eliminate the outlier data. In particular, for $\hat{M} = 0, 1$, and $2N - 1$ we use the whole data set $w = x$, for $\hat{M} \geq N + 1$ we use $w = y$ otherwise use $w = x$. Since the outlier/misleading data are eliminated from data with high probability, we expect that the elements of $w$ have iid distributions characterizing the clutter component of the signal $x_{\text{CUT}}$ at the CUT. We propose the following detector which is based on the log-detector,

$$x_{\text{CUT}} = H_1 \frac{H_1}{H_0} \frac{\text{std}(w)}{T_{L_{\hat{M}}}} + \overline{w}. \quad (16)$$

where $\overline{w} = \frac{1}{\hat{M}} \sum_{w_k \in w} w_k$ and $L_{\hat{M}}$ are the sample mean and length of $w$ respectively. The hypotheses $H_1$ and $H_0$ denote the presence and the absence of a target in the CUT, respectively. The selected data length varies depending on the edge location, i.e., $L_{\hat{M}} = 2N$ for $H_0$, $H_1$ and $H_{2N-1}$. $L_{\hat{M}} = \max(2N - \hat{M}, \hat{M})$ otherwise. The constant threshold $T_{L_{\hat{M}}}$ is related to $L_{\hat{M}}$ and it is set to satisfy the desired false alarm rate $P_{fa}$. To implement this CFARD we must compute all constant values of $T_{L_{\hat{M}}}$ for all possible values of $L_{\hat{M}} \in \{N, \ldots, 2N\}$ in advance. In [17] for a homogeneous Weibull clutter environment without any edge, the relation between the probability of false alarm $P_{fa}$ and the detection threshold is investigated. The proposed detector given by (16) and the log-det CFARD in [17] have the same expression. Their difference is that the size of the reference data which in our method varies depending on the estimated edge location. Thus, for very large $N$ the asymptotical results and curves in [17] may be used to set our threshold values. In this paper, we used computer simulation to pre-calculate (off-line) the values of $T_{L_{\hat{M}}}$ for $L_{\hat{M}} \in \{N + 1, \ldots, 2N\}$ for given desired $P_{fa}$. Figure 4 shows the curves of detection threshold $T_{L_{\hat{M}}}$ versus nominal $P_{fa}$ for different data length $L_{\hat{M}}$ from 16 to 32. Figure 5 presents the curves of $T_{L_{\hat{M}}}$ versus $L_{\hat{M}}$ for various nominal values for $P_{fa}$ from $10^{-6}$ to $10^{-2}$.

**Lemma 1**: The detector in (16) is a CFARD in a homogeneous environment.

**Proof**: We can rewrite (16) as $t = \frac{x_{\text{CUT}} - \overline{w}}{\text{std}(w)} \frac{H_1}{H_0} T_{L_{\hat{M}}}$ from $L_{\hat{M}} = 16$ to $L_{\hat{M}} = 32$. The estimated edge location $\hat{M}$ is random. However, we proceed our proof by substituting a given possible value for $\hat{M}$. We consider the following transformation group

$$G_{\alpha, \beta} = \left\{ g_{\alpha, \beta} \left( x_{\text{CUT}} \right) = \alpha x + \beta, \alpha \geq 0, \beta \in \mathbb{R}, x \in \{ x_{\text{CUT}}, w \} \right\} \quad (17)$$

applied on the data, where $\alpha$ and $\beta$ are scale and shift parameters, respectively. Under $H_0$ our data set $\{ x_{\text{CUT}}, w \}$ has an iid Gumbell distribution. It is easy to show that the data transformed by group $G_{\alpha, \beta}$ has also an iid Gumbell distribution. In addition, the ensemble statistics of the transformed data $g_{\alpha, \beta}(w)$ are $g_{\alpha, \beta}(w) = \alpha w + \beta$ and $\text{std}(g_{\alpha, \beta}(w)) = \alpha \text{std}(w)$. Interestingly, by substituting $g_{\alpha, \beta}(x_{\text{CUT}})$, $g_{\alpha, \beta}(w)$ and $\text{std}(g_{\alpha, \beta}(w))$ into the test statistics $t$, we observe that the test statistics $t$ is invariant and is not a function of $\alpha$ and $\beta$. Since by selecting $\{ \alpha, \beta \}$ we set the parameters of the Gumbell distribution of $g_{\alpha, \beta}(\{ x_{\text{CUT}}, w \})$ to any arbitrary permissible value, we conclude that the false alarm probability does not depend on the parameters of the distribution. In other words, for a given $\hat{M}$, this detector is CFAR against the variation of clutter parameters, i.e., the conditional false alarm probability $P[t > T_{L_{\hat{M}}} | \hat{M}] = P_{fa}$ is constant with respect to the parameters of the environment. The detection thresholds $\{ T_{L_{\hat{M}}} \}_{\hat{M}=0}^{2N-1}$ are set such...
that the conditional false alarm probability \( P[t > T_{L,n} | M] \) is fixed to a pre-specified value \( P_{fa} \) for all \( M \), i.e., \( P[t > T_{L,n} | M] \) is invariant with respect to parameters of the environment. We now prove that the distribution of \( M \) selected in (14) and (15) remains invariant under the transformation group \( G_{\alpha, \beta} \). If we define \( T(x, M) = \{(2N - M) \ln(\text{std}(x_M)) + M \ln(\text{std}(x_M))\} - 2N \ln(\text{std}(x)) \), the selections in (14) and (15) are respectively equivalent to maximizing \( T(x, M) \) over \( M = 2, \cdots, 2N - 2 \) and comparing \( T(x, M) \) with \( \xi \) (i.e., \( -\xi \) is added only for \( H_0 \)). From \( \ln[\text{std}(\alpha + \beta)] = \ln \alpha + \ln[\text{std}(x)] \), it is easy to see that \( T(x) = T(\alpha + \beta) \). Since the transform \( \alpha + \beta \) allows to make any arbitrary change in the parameters of our assumed homogenous clutter, the distribution of \( T(x) \) does not depend on the parameters of the clutter. This means that the distribution of \( M \), i.e., \( P[M] \), does not depend on the clutter parameters. We conclude that the overall unconditional false alarm probability is also constant, i.e., \( \sum_{n=M} P[t > T_{L,n} | M] P[M] = P_{fa} \) and invariant with parameters of the environment.

IV. SIMULATION RESULTS

In this section, we evaluate and compare, by computer simulations the performance of proposed CFARD with that of three well-known detectors namely, log-t, WH-CFAR and ML-CFAR. As a benchmark, we also compare the performance of the proposed CFARD with that of following optimal detector

\[
x_{\text{CUT}} \overset{H_0}{=} T_{\text{opt}} = b + a \ln \left( \frac{1}{P_{\text{fa}}} \right)
\]

where the threshold \( T_{\text{opt}} \) is a function of the Gumbel clutter parameters \( (a, b) \) on the CUT and \( P_{fa} \) is the nominal false alarm rate. We must note that this optimal detector requires the exact values of parameters of Gumbel (Weibull) clutter at the CUT. Since the parameters \( a \) and \( b \) are unknown, this detector is not implementable in practice. However, we use it as a benchmark reference for the implementable detectors.

In WH CFARD [24], the reference samples are sorted in ascending order as \( z_1 \leq z_2 \leq \cdots \leq z_{2N} \). Then, the detection threshold \( T_{\text{WH}} = z_{k_1} - \beta z_{k_2} \) is calculated using the \( k_1^{th} \) and \( k_2^{th} \) order samples \( z_{k_1}, z_{k_2} \) (\( 1 \leq k_1 \leq k_2 \leq 2N \)) where

\[
\beta = \frac{\ln(\text{std}(\alpha_{k_1})) - \ln(\text{std}(\alpha_{k_2}))}{\ln(1 - \frac{k_1}{2N + 1}) - \ln(1 - \frac{k_2}{2N + 1})}
\]

and \( \alpha_{k_i} \) is the solution of the following non-linear equation for nominal \( P_{fa} \)

\[
P_{fa} = \frac{(2N)!((\alpha_{k_1} + 2N - k_1))!(\alpha_{k_2} + 2N)!}{(2N - k_1)!(\alpha_{k_1} + 2N)!}. \tag{20}
\]

In ML CFARD, the detection threshold is calculated using the MLE of Gumbel (Weibull) parameters of clutter on the CUT as follows

\[
T_{\text{ML}} = \hat{b}_{\text{ML}} + \hat{\alpha}_{\text{ML}} \ln(\alpha)
\]

where \( \hat{\alpha}_{\text{ML}} \) and \( \hat{b}_{\text{ML}} \) are the solution of non-linear equations in (3) and (4) calculated iteratively and \( \alpha \) is a constant coefficient which is set to satisfy a nominal \( P_{fa} \) [18].

We consider a window size of \( 2N = 32 \) and generate independent random samples of clutter with Weibull distribution. For the CUT, the received signal has a clutter component plus a target signal component generated following Swerling I model [1] with a normalized random Doppler shift uniformly distributed over \((-\pi, \pi]\). Throughout our simulations, the nominal probability of false alarm has been set to \( 10^{-5} \) except in Figure 6. For the WH-CFAR detector we set the ranks \( k_1 = 5 \) and \( k_2 = 29 \) (see [24] for more details). We use an extrapolative estimator for computing small probabilities (< \( 10^{-4} \)) which is based on generalized extreme-value theory [38], [39].

First we present and compare the performance of proposed algorithm for values of \( \lambda \) around \( 2 \), where \( \xi = \lambda \ln(2N) \) in (16). We start with \( \lambda \) around \( 2 \) since the difference between the numbers of unknown parameters under \( H_0 \) and any other hypotheses, is \( 2 \) [36], [37]. Thus in Figure 7, we evaluate the proposed detector for \( \lambda \in [1, 3] \), where \( P_{fa} \) is plotted versus the signal to clutter ratio (SCR) for \( B = 1 \) and various values of \( C \). We see that in the homogenous situation, the smaller values of \( \lambda \) result in serious degradation in the detection performance. For all \( \lambda \geq 2 \), the performance of the proposed detector is very close to that of log-t. Figure 8 shows \( P_{fa} \) (curves below) and \( P_{\alpha} \) (curves above) of the proposed detector versus the clutter edge position for \( \lambda \in [1, 3] \). Both parameters of clutter are different in both sides of the clutter edge, (here \( [B_y, C_y] = [1, 7] \) and \( [B_z, C_z] = [10, 1.6] \) are parameters of clutter in the first and second partitions of reference samples, respectively). This figure illustrates the ability of the proposed detector in maintaining the false alarm rate around its nominal value as the clutter edge changes. For a smaller \( \lambda \), it is more likely to detect an edge. However, the price is more false alarms in the edge detection. Thus, we must adjust \( \lambda \) to make a reasonable trade off between the performance for the cases without
edge and for the cases where the edges are more frequent. We see that the false alarm and the detection probabilities exhibit degradation for $\lambda > 2$ for some edge locations. In contrast, $\lambda \leq 2$ results in an acceptable performance. Therefore, based on the results of Figure 7 and Figure 8, we hereafter use $\lambda = 2$ (i.e., we recommend to set $\lambda = 2$ or around 2) as a reasonable choice.

In Figure 9, the performance of proposed detector is compared with that of log-t, WH, ML and optimal CFARs in homogenous clutter environment for different values of $C$. As expected, in a homogenous environment, the performance of the proposed detector is almost the same as that of log-t for various values of $SCR$ and $C$. These two detectors are generally better than WH CFARD especially for $C = 2$ (note that $C = 2$ represents the case of Gaussian clutter). The ML CFARD has the best performance after the optimal CFARD.

Figure 10 shows that the proposed detector significantly outperforms the log-t, WH and ML CFARs in the presence of a clutter edge. This figure illustrates the curves of $P_{fa}$ and $P_d$ versus clutter edge position where the parameters of clutter are $B_y, C_y = [1, 0.7]$, $[B_z, C_z] = [10, 1.6]$ for the first and second partitions, respectively. For $P_d$ curve, the $SCR$ is set to 10dB. In this figure, the considerable degradation of ML-CFARD is meaningful. Figures 11a and 11b show similar curves, where either the shape parameter or scale parameter is identical in both sides of the clutter edge, respectively. These figures show that in the presence of a clutter edge in the reference window, the proposed detector adjusts the actual $P_{fa}$ around the nominal value and outperforms in terms of the detection probability compared with other detectors, especially when both parameters of the clutter are different in both sides of the clutter edge.

In many of figures, e.g. in Figure 10, we observe that when a clutter edge enters the reference window, the false alarm rate and the detection probabilities exhibit considerable variation as the edge approaches the CUT. The reason is that these detectors (excluding the optimal and the proposed CFARDs) use all reference samples to determine the detection threshold. In such a case, a portion of reference samples includes the outlier data and impacts the threshold and consequently results in the deviation of $P_{fa}$ from its nominal value and the degradation of $P_d$. When the edge is located around the CUT, the probability that the proposed detector mistakenly detects CUT in the outlier side increases. This event can happen if the edge detection error is such that the CUT is mistakenly detected in the outlier region. In such an event, the outlier data is mistakenly used for the calculation of the detection threshold. As a result, the curves of $P_{fa}$ and $P_d$ for the proposed detector exhibit sudden variations as the clutter edge tends to the CUT. The amount of the variations depend on the change of parameters in two sides of the clutter edge.

Figure 12 compares the detection probability $P_d$ of the proposed detector with that of other mentioned detectors versus SCR in the presence of clutter edge. In Figure 12a, the clutter edge is located between the 20th and 21st reference cells and the nominal false alarm rate $P_{fa}$ is set to $10^{-3}$. In this example, the superiority of the proposed detector is quite perceptible, in particular as the clutter distribution approaches the Gaussian distribution. In Gaussian clutter, the proposed detector outperforms the log-t and WH-CFARDs respectively with 23dB and 28dB gains in SCR. In Weibull Clutter for $C = 1$, these SCR improvements are 10dB and 15dB respectively. In Figure 12b, the clutter edge is located between the 16th and 17th reference cells (i.e. in the center of reference window and on the CUT), and the nominal false alarm rate $P_{fa}$ is $10^{-3}$. We see that the performances of all of the implementable CFARs considerably degrade in comparison with the optimal non-implementable CFARD, because of this special clutter edge location. However, the proposed CFARD outperforms other existing methods and by far its performance is closer to that of optimal non-implementable CFARD.
compare the Receiver Operating Characteristic (ROC) of the proposed

and behaves more similar to the optimal detector. In Figure 15, we

nominal

P

to Gaussian.

WH CFARD becomes superior, i.e., as the clutter distribution tends

to improve the target detection performance in a non-homogeneous

We present an example in Figure 13 to evaluate the performance

of the proposed detector against an interfering target in the reference

window. In Figure 13, \( P_a \) is plotted versus SCR, where an interfering
target is present in the 10\(^{th} \) reference cell with 20dB Interference to

Clutter Ratio (ICR). The Weibull clutter parameters are \( B = 1 \) and

\( C \) either 0.5, 1, 1.5 or 2. We observe that the proposed detector

exhibits the same performance as log-t CFARD in various cases of clutter. For \( C = 0.5 \) (which is far away from the Gaussian case), the proposed
CFARD, log-t, and ML CFARD outperform the WH CFARD, respectively. In contrast as \( C \) increasingly tends to 2, the WH CFARD becomes superior, i.e., as the clutter distribution tends to Gaussian.

To illustrate that our results are conclusive, regardless of the

nominal \( P_{fa} \), we regenerated Figure 9 for \( P_{fa} = 10^{-5} \) instead of

\( P_{fa} = 10^{-3} \). In this case, our results show similar trends for various
cases. As example, Figure 6 illustrates the curves for \( C = 0.5 \) and

2. From these results, we conclude that the obtained results are not

sensitive to the nominal value of \( P_{fa} \).

Figure 14 illustrates the curves of \( P_a \) and \( P_i \) versus the clutter

power ratio between two clutter parts CCR for \( C_y = 0.7, C_z = 1.6, \)

\( B_y = 10 \), nominal \( P_a = 10^{-3} \), and a clutter edge located at 10. We see

that the proposed detector outperforms other non-optimal CFARDs

and behaves more similar to the optimal detector. In Figure 15, we

compare the Receiver Operating Characteristic (ROC) of the proposed
detector with that of log-t, WH, ML and optimal CFARDs, for \( B_y = 1, B_z = 10 \) and for three values of shape parameters \( C_y = C_z = 1, 1.5, 2 \) where a) the clutter edge is between the 20\(^{th} \) and 21\(^{th} \) reference cells, b) the clutter edge is between the 16\(^{th} \) and 17\(^{th} \) reference cells. 2\( N = 32 \).

We present an example in Figure 13 to evaluate the performance

of the proposed detector against an interfering target in the reference

window. In Figure 13, \( P_a \) is plotted versus SCR, where an interfering
target is present in the 10\(^{th} \) reference cell with 20dB Interference to

Clutter Ratio (ICR). The Weibull clutter parameters are \( B = 1 \) and

\( C \) either 0.5, 1, 1.5 or 2. We observe that the proposed detector

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CFARD, log-t, and ML CFARD outperform the WH CFARD, respectively. In contrast as \( C \) increasingly tends to 2, the WH CFARD becomes superior, i.e., as the clutter distribution tends to Gaussian.

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V. CONCLUSION

In this paper, we proposed an edge detection algorithm in order to

improve the target detection performance in a non-homogeneous

Weibull clutter environment. Indeed, this detector results in significant
improvement around the clutter edges. This pre-processing algorithm

eliminates the outlier data and feeds the clutter data into a threshold
calculator for the target detection. We proved that the proposed
detector has a constant false alarm rate against variations of all

unknown parameters of a Weibull clutter. Our results reveal that the

performance of the proposed detector is the same as that of the log-t
detector in homogeneous environment and in presence of interfering
target. However, the proposed detector significantly outperforms the

log-t and the WH-CFAR detectors if there is a clutter edge in

the reference window, i.e., it is a superior detector compared with

log-t and WH-CFAR detectors, particularly in non-homogeneous

environments.
Fig. 13. Detection performance of the proposed detector compared with log-t, WH, ML and optimal CFARDs in the presence of an interfering target with ICR=20 dB at the 10\textsuperscript{th} cell; for $C_y=0.7$, $C_x=1.6$, $B_x=10$, nominal $P_{fa}=10^{-5}$ and $2N=32$. a) Probability of false alarm, b) Probability of detection for SCR=15 dB.

Fig. 14. False alarm rate and Detection performance of the proposed detector versus CCR compared with log-t, WH, ML and optimal CFARDs, for $C_y=0.7$, $C_x=1.6$, $B_x=10$, nominal $P_{fa}=10^{-5}$ and $2N=32$. a) Probability of false alarm, b) Probability of detection for SCR=10 dB.

Fig. 15. $P_{fa}$ versus $P_d$ (ROC) of the proposed detector compared with log-t, WH and optimal CFARDs for $[B_x, C_y]=[1, 0.7]$, $[B_x, C_y]=[10, 1.6]$ and $2N=32$. a) Homogenous clutter, clutter edge at 0, and SCR=0, 7.5, 15 dB, b) non-homogenous clutter, clutter edge at 20, and SCR=15, 40 dB.

REFERENCES


